

Contents

The Loop Formulation of Gauge Theory and Gravity	1
<i>Renate Loll</i>	
1 Introduction	1
2 Yang–Mills theory and general relativity as dynamics on connection space	1
3 Introducing loops!	6
4 Equivalence between the connection and loop formulations	8
5 Some lattice results on loops	11
6 Quantization in the loop approach	13
Representation Theory of Analytic Holonomy C^*-algebras	21
<i>Abhay Ashtekar and Jerzy Lewandowski</i>	
1 Introduction	21
2 Preliminaries	26
3 The spectrum $\overline{\mathcal{A}/\mathcal{G}}$	30
4 Integration on $\overline{\mathcal{A}/\mathcal{G}}$	39
5 Discussion	49
Appendix A: C^1 loops and $U(1)$ connections	51
Appendix B: C^ω loops, $U(N)$ and $SU(N)$ connections	56
The Gauss Linking Number in Quantum Gravity	63
<i>Rodolfo Gambini and Jorge Pullin</i>	
1 Introduction	63
2 The Wheeler–DeWitt equation in terms of loops	67
3 The Gauss (self-)linking number as a solution	71
4 Discussion	73
Vassiliev Invariants and the Loop States in Quantum Gravity	77
<i>Louis H. Kauffman</i>	
1 Introduction	77
2 Quantum mechanics and topology	78
3 Links and the Wilson loop	78
4 Graph invariants and Vassiliev invariants	87
5 Vassiliev invariants from the functional integral	89
6 Quantum gravity—loop states	92

Geometric Structures and Loop Variables in (2+1)- Dimensional Gravity	97
<i>Steven Carlip</i>	
1 Introduction	97
2 From geometry to holonomies	98
3 Geometric structures: from holonomies to geometry	104
4 Quantization and geometrical observables	107
From Chern–Simons to WZW via Path Integrals	113
<i>Dana S. Fine</i>	
1 Introduction	113
2 The main result	114
3 $\mathcal{A}/\mathcal{G}_n$ as a bundle over $\Omega^2 G$	115
4 An explicit reduction from Chern–Simons to WZW	118
Topological Field Theory as the Key to Quantum Gravity	121
<i>Louis Crane</i>	
1 Introduction	121
2 Quantum mechanics of the universe. Categorical physics	123
3 Ideas from TQFT	125
4 Hilbert space is dead—long live Hilbert space or the observer gas approximation	128
5 The problem of time	129
6 Matter and symmetry	130
Strings, Loops, Knots, and Gauge Fields	133
<i>John C. Baez</i>	
1 Introduction	133
2 String field/gauge field duality	136
3 Yang–Mills theory in 2 dimensions	142
4 Quantum gravity in 3 dimensions	153
5 Quantum gravity in 4 dimensions	158
BF Theories and 2-knots	169
<i>Paolo Cotta-Ramusino and Maurizio Martellini</i>	
1 Introduction	169
2 <i>BF</i> theories and their geometrical significance	170
3 2-knots and their quantum observables	175
4 Gauss constraints	177
5 Path integrals and the Alexander invariant of a 2-knot	180
6 First-order perturbative calculations and higher-dimensional linking numbers	181
7 Wilson ‘channels’	182

8	Integration by parts	184
	Knotted Surfaces, Braid Movies, and Beyond	191
	<i>J. Scott Carter and Masahico Saito</i>	
1	Introduction	191
2	Movies, surface braids, charts, and isotopies	192
3	The permutohedral and tetrahedral equations	204
4	2-categories and the movie moves	214
5	Algebraic interpretations of braid movie moves	218