

Contents

List of contributors	xiii
Guaranteed Error Bounds for Ordinary Differential Equations	1
<i>George F. Corliss</i>	
1 Introduction	2
2 Requirements: validated solutions	4
2.1 What is a “validated solution” of an ODE?	4
2.2 When are validated solutions needed?	6
3 Feasibility	8
3.1 Existence and uniqueness	9
3.2 Enclosures	11
3.3 When we are doomed to fail	14
4 Tools	16
4.1 Interval arithmetic	17
4.2 Automatic differentiation	31
5 Specifications: get tight intervals	34
5.1 Maple worksheet tighter.ms	35
5.2 Approximation + [Error]	44
5.3 Wrapping effect	46
6 Design	46
6.1 Differential inequalities	46
6.2 Defect-controlled solutions	48
6.3 Lohner’s method	56
7 Conclusions and further research directions	69
Acknowledgments	70
Bibliography	70
Introduction to Computational Methods for Differential Equations	77
<i>Kenneth Eriksson, Don Estep, Peter Hansbo and Claes Johnson</i>	
1 Leibniz’s vision	78
2 The discretization method	79
3 An introductory example	80
4 Numerical methods	83
4.1 Reliability and efficiency	83
4.2 A priori and a posteriori error estimates	84
4.3 Accuracy and stability	84

4.4	The total error	85
4.5	Femlab	86
5	Piecewise polynomial approximation in one dimension	86
5.1	Discontinuous piecewise constant approximation	86
5.2	Continuous piecewise linear approximation	88
5.3	Galerkin approximations	90
6	An elliptic model problem	90
6.1	An a posteriori error estimate	92
6.2	An adaptive algorithm	93
6.3	An a priori error estimate	94
6.4	Dirichlet and Neumann boundary conditions	95
6.5	Modeling and data errors	95
7	A “parabolic” model problem	96
7.1	An a posteriori error estimate	97
7.2	Adaptive error control	100
7.3	An a priori error estimate	101
7.4	The effect of quadrature	104
8	A “hyperbolic” model problem	106
8.1	An a posteriori error estimate	106
8.2	An a priori error estimate	107
9	The general initial value problem	109
9.1	An a posteriori error estimate	109
10	An elliptic PDE: Poisson’s equation	112
10.1	Introduction	112
10.2	Poisson’s equation	113
10.3	Fem for Poisson’s equation	114
10.4	Estimates of the interpolation error	115
10.5	A priori error estimates in the energy norm	115
10.6	A posteriori error estimates in the energy norm	115
11	A parabolic PDE: the heat equation	116
12	Concluding remarks. References	117
13	How to obtain Femlab and Introduction to Computational Methods for Differential Equations	118
	Bibliography	119
Numerical Solution of Differential-Algebraic Equations		
<i>Linda R. Petzold</i>		123
1	Introduction	123
2	Structure of DAE systems	124
3	Multistep methods	126
4	Runge–Kutta methods	128
5	Software for DAEs	131
6	Applications	135

Bibliography	138
Boundary Element Methods	143
<i>Ian H. Sloan</i>	
1 A first brief tour of boundary element methods	143
1.1 Introduction	143
1.2 A model problem	144
1.3 A first boundary integral equation	144
1.4 Two important numerical methods — the Galerkin and collocation methods	145
2 Basic boundary integral equations	148
2.1 Classical BIE formulations	148
2.2 The classical BIE quartet	150
2.3 Direct method	153
2.4 Three-dimensional problems	154
3 Smoothing properties, Sobolev spaces, and so forth	155
3.1 Introduction	155
3.2 The single-layer operator	155
3.3 Other integral operators	160
3.4 Regions with corners	160
4 Strong ellipticity and the Galerkin method	161
4.1 Introduction	161
4.2 Strong ellipticity	161
4.3 The Galerkin method	163
5 Assorted methods for a planar problem	164
5.1 Introduction	164
5.2 Galerkin method	165
5.3 The collocation method	167
5.4 The qualocation method	168
5.5 A sketch of the proof of Theorem 5.1	170
5.6 Higher-order convergence	173
5.7 The qualocation method for a region with corners	175
Bibliography	177
Perturbation Theory for Infinite Dimensional Dynamical Systems	181
<i>Andrew Stuart</i>	
1 Introduction	182
2 Evolution equations in a Hilbert space	186
2.1 Introduction	186
2.2 Sectorial operators	186
2.3 Notation	189
2.4 The evolution equation	190
2.5 Sectorial evolution equations	190

2.6	The Navier–Stokes equations	196
2.7	The Cahn–Hilliard equation	196
2.8	Ordinary differential equations	197
2.9	Semigroups	197
2.10	Bibliography	202
3	Basic approximation of trajectories	202
3.1	Introduction	202
3.2	Approximation assumptions	203
3.3	Spectral method for the sectorial equation	205
3.4	Backward Euler method for sectorial equations	211
3.5	The phase-field and viscous Cahn–Hilliard equations	211
3.6	Ordinary differential equations	212
3.7	Bibliography	212
4	Equilibria and phase portraits	213
4.1	Introduction	213
4.2	Equilibria	213
4.3	Trajectories asymptotic to a stable steady state	218
4.4	Phase portraits near a saddle	223
4.5	Bibliography	233
5	Unstable manifolds	234
5.1	Introduction	234
5.2	Background theory	234
5.3	Local unstable manifolds	236
5.4	Global unstable sets	241
5.5	Bibliography	242
6	Inertial manifolds	243
6.1	Introduction	243
6.2	Existence theory	243
6.3	Perturbation theory	249
6.4	Bibliography	253
7	Attractors	253
7.1	Introduction	253
7.2	Background theory	254
7.3	Upper-semicontinuity of attractors	258
7.4	Continuity for exponentially attracting attractors	259
7.5	Lower-semicontinuity of attractors	261
7.6	Bibliography	262
8	Error analysis for gradient systems	263
8.1	Introduction	263
8.2	The result	264
8.3	Bibliography	270
9	Practical numerical stability	270
	Bibliography	271

10	Appendix A — Sectorial evolution equations	278
11	Appendix B — Contraction principles and Taylor expansions	284
12	Appendix C — Attractive invariant manifolds	286
Delay Differential Equations: Theory and Numerics		291
<i>M. Zennaro</i>		
1	Introduction	291
2	Theoretical analysis of a class of delay differential equations	292
2.1	Location of discontinuities and existence of solutions	293
2.2	Some important particular equations	293
2.3	Stability	294
3	How to solve delay differential equations numerically	298
3.1	A general approach	299
3.2	Runge–Kutta methods and one-step interpolation	300
3.3	Runge–Kutta methods for delay differential equations	303
3.4	Available methods and software	307
4	Stability properties of numerical methods	308
4.1	P-stability and GP-stability	308
4.2	PN-stability and GPN-stability	315
4.3	RN-stability and GRN-stability	319
4.4	Further stability investigations	320
5	More general classes of delay differential equations	321
5.1	Equations with vanishing delay	321
5.2	Neutral delay differential equations	324
5.3	Equations with state dependent delays	326
5.4	Equations with several delays	327
Bibliography		328