

Contents

Preface	xii
---------	-----

Lecture Series 1. Infinite Matrices and Projection Methods

Albrecht Böttcher

1. Matrix representations of operators	3
2. Three problems for infinite matrices	7
3. The finite section method	14
4. Selfadjoint and compact operators	16
5. Toeplitz matrices with continuous symbols	20
6. Toeplitz operators: algebraization of stability	25
7. Toeplitz operators: localization	30
8. Block case and higher dimensions	35
9. Banach space phenomena	43
10. Norms of inverses and pseudospectra	48
11. Toeplitz determinants	59
12. More general projection methods	64
Bibliography	69

Lecture Series 2. Operator Theory and Ordinary Differential Operators

Aad Dijksma and Heinz Langer

Introduction	75
1. Definitizable operators in Kreĭn spaces	77
1. Basic concepts	77
2. Definitizable operators	80
3. The spectral function of a definitizable operator	82
4. Critical points	84
5. The critical point ∞	85

2. Boundary eigenvalue problems for Sturm-Liouville operators and related holomorphic functions	87
1. The Sturm–Liouville differential operator	87
2. The Titchmarsh–Weyl coefficient	90
3. The boundary eigenvalue problem	93
4. The linearization of the boundary eigenvalue problem	94
5. Special cases	96
3. Operator representations of holomorphic functions	99
1. Colligations	99
2. Linearizations and Štraus extensions	101
3. Models for the Titchmarsh–Weyl coefficients	106
4. Sturm–Liouville operators with indefinite weight	109
1. The problem	109
2. Full range and half range completeness	113
3. The Titchmarsh–Weyl coefficient	115
5. Interface conditions and singular potentials	117
1. Interface conditions	117
2. A singular potential continuous in the graph norm of \hat{A}	120
3. A singular potential discontinuous in the graph norm of \hat{A}	122
4. Strings with point masses and dipoles	125
6. Operator pencils	128
1. Basic notations	128
2. Operator polynomials	128
3. A λ -rational pencil	130
4. The Klein–Gordon equation	131
Bibliography	133
Symbols used in Lecture Series 2	139
Lecture Series 3. Operators on Indefinite Inner Product Spaces	
Michael A. Dritschel and James Rovnyak	
Preface	143
Introduction: Preliminaries and notation	144
A. Prerequisites	144
B. Inner product spaces	144
C. Kreĭn spaces and fundamental decompositions	145
D. Example	147
E. Books and expositions	148
Lecture 1. Kreĭn spaces and operators	149

Lecture 2. Julia operators and contractions	158
Addendum: Adjoints of contractions	168
Lecture 3. Extension and completion problems	171
Lecture 4. The Schur algorithm	181
Lecture 5. Reproducing kernel Pontryagin spaces and colligations	192
Lecture 6. Invariant subspaces	207
A. The commutant of a selfadjoint operator	207
B. Dual pairs of invariant subspaces for definitizable operators	213
C. Compact perturbations of definitizable operators	221
Addendum: Nondefinitizable operators	224
Bibliography	227
Lectures Series 4. State Space Theory of Rational Matrix Functions and Applications	
M.A. Kaashoek	
Introduction	235
Chapter 1. Canonical factorization and the state space method	237
1.1. Elements of mathematical system theory	237
1.2. Inverting systems of Wiener–Hopf integral equations	242
1.3. Canonical factorization of rational matrix functions	244
1.4. Wiener–Hopf integral equations revisited	248
1.5. Notes	250
Chapter 2. J -unitary rational matrix functions	252
2.1. J -unitary rational matrix functions	252
2.2. J -inner rational matrix functions	253
2.3. J -inner-outer factorization in $\mathcal{Rat}(H_\infty)$	258
2.4. Notes	262
Chapter 3. Analysis of zeros	263
3.1. Eigenvectors and canonical systems of null functions	263
3.2. Canonical right and left null pairs at a point	268
3.3. Right and left null pairs on a set	271
3.4. The analytic case	274
3.5. Notes	276
Chapter 4. Inverse problems involving null pairs	277
4.1. An inverse problem in $\mathcal{Rat}(H_\infty)$	277
4.2. J -inner matrix functions in $\mathcal{Rat}(H_\infty)$ with prescribed null pairs	279
4.3. Left tangential Nevanlinna–Pick interpolation	281
4.4. Notes	287

Chapter 5. Analysis of zeros and poles	288
5.1. Pole pairs and null-pole subspace	288
5.2. Coupling transformation and null-pole triples	290
5.3. More about the coupling transformation	294
5.4. Null-pole triples from minimal realizations	297
5.5. Another construction of a null-pole triple	299
5.6. Null-pole triples and the Smith–McMillan form	304
5.7. Notes	308
Chapter 6. Inverse problems involving null-pole triples	309
6.1. Rational matrix functions with prescribed null-pole triples	309
6.2. J -inner matrix functions with prescribed null-pole triples	316
6.3. Bitangential Nudelman interpolation	320
6.4. Notes	324
Bibliography	327
Index	335