Table of contents

Welcoming remarks XIII

Editorial introduction XVII

Vladimir Petrovich Potapov, as remembered by colleagues, friends and former students 1

M.S. Livšic 1
D.Z. Arov 8
L.A. Sakhnovich 12
A.A. Nudelman 13
V.K. Dubovoj 14
V.E. Katsnelson 16

N.I. Akhiezer

On a minimum problem in function theory and the number of roots of an algebraic equation inside the unit disc 19

0. Introduction 19
1. An auxiliary result 20
2. The Carathéodory-Fejér problem 23
3. A remark on Equation (14) 28
4. An application to the problem on the number of roots of an algebraic equation in the disc 29
5. The G. Pick problem 31

References 34

D. Alpay and V. Bolotnikov

On tangential interpolation in reproducing kernel Hilbert modules and applications 37

1. Introduction 37
2. Reproducing kernel Hilbert modules 39
3. The left-sided Nevanlinna-Pick problem 43
4. The left-sided Carathéodory-Fejér problem 46
5. Right-sided interpolationlj 49
6. On the recursive solution and the solvability criterion 52
7. The case of Hardy spaces 54
8. An example: interpolation in Hardy-Sobolev spaces 54

9. Interpolation in Dirichlet spaces 58
10. Interpolation in Bergman spaces 59
11. Some remarks on interpolation in de Branges-Rovnyak spaces 60
References 65

A. Dijksma and H. Langer

Notes on a Nevanlinna-Pick interpolation problem for generalized Nevanlinna functions 69

1. Introduction 69
2. The model 72
3. Selfadjoint extensions of the model and the solutions of the interpolation problem 77
4. A parametrization 79
5. Excluded parameters 84
References 89

V.K. Dubovoj

The indefinite metric in the Schur interpolation problem for analytic functions, IV 93

10. Solving the degenerate Schur problem 93
References 103

H. Dym and B. Freydin

Bitangential interpolation for upper triangular operators 105

1. Introduction 105
2. The basic interpolation problem 110
3. The augmented BIP 115
4. Coupling 125
5. Linear fractional representation of all solutions 132
6. Formulas for the standard unitary colligation A 137
References 141

H. Dym and B. Freydin

Bitangential interpolation for upper triangular operators when the Pick operator is strictly positive 143

7. Strictly positive Pick operators 143
8. Chain scattering operators 150
9. The maximum entropy problem 157
10. Positive real interpolants 159
References 163

Yu.M. Dyukarev

Integral representations of a pair of nonnegative operators and interpolation problems in the Stieltjes class 165

1. Introduction 165
2. The formulation of the problem about consistent representation of the pair of nonnegative operators 166
3. The system of fundamental matrix inequalities of V.P. Potapov 168
4. The transformation of the FMI of V.P. Potapov 171
5. J-expansive functions associated with a nondegenerate system of FMI's 177
6. The solution of the FMI system in the nondegenerate case 181

References 183

Yu.P. Ginzburg

On recovering a multiplicative integral from its modulus 185

0. Preface 185
1. On one factorization of an absolutely continuous operator function 186
2. The modulus of the multiplicative integral of a Hermitian-valued function 187
3. On some classes of analytic operator functions and their multiplicative representations 188

References 193

L. Golinskii

On Schur functions and Szegö orthogonal polynomials 195

References 203

L. Golinskii and I. Mikhailova

Hilbert spaces of entire functions as a J theory subject 205

0. Preface 205
1. Preliminaries from J theory 206
2. de Branges matrices 212
3. Parametrization of de Branges matrices 220
4. de Branges spaces 226
5. Structure of perfect de Branges matrices and basic formulas 229
6. Parseval's equality 235

References 251

V.E. Katsnelson

On transformations of Potapov's fundamental matrix inequality 253

0. Preface 253
1. The FMI and its structure 254
2. The FMI for the Nevanlinna-Pick problem 256
3. Derivation of the FMI ($\mathcal{NP}$) 257
4. The Hamburger moment problem as a classical interpolation problem 260
5. Derivation of the FMI ($\mathcal{H}$) 263
6. Transformation of the FMI ($\mathcal{H}$) 265
7. On using the TFMI ($\mathcal{H}$) to obtain interpolation information from the FMI ($\mathcal{H}$) 269
8. Transformation of the FMI ($\mathcal{NP}$) 274

References 280

V.E. Katsnelson, A.Ya. Kheifets and P.M. Yuditskii

An abstract interpolation problem and the extension theory of isometric operators 283

1. Interpolation data and examples 283
2. V.P. Potapov's fundamental matrix inequality and its transformation 285
3. The abstract interpolation problem 295

References 297

V.E. Katsnelson and B. Kirstein

On the theory of matrix-valued functions belonging to the Smirnov class 299

0. Preface 300
1. On the matricial Nevanlinna and Smirnov classes 302
2. Matrix functions of the Smirnov class as multiples of contractive matrix functions 307
3. Outer matrix-valued functions 310
4. Matrix-valued inner functions 319
5. Inner-outer factorization 323
6. An analogue of Frostman's theorem for matrix functions of the Smirnov class 330

References 341

I.V. Kovalishina

Integral representation of function of class $\mathcal{K}_a$ 351

1. Formulation of the problem 351
2. The fundamental matrix inequality (FMI) 353
3. The unification of the FMI and the Sakhnovich identity 355
4. The case of more than one representation 356

References 359

M.G. Krein

On the theory of entire matrix-functions of exponential type 361

0. Introduction 361
1. Proof of the main theorem 362
2. Applications of the theorem for the investigation of a monodromy matrix 364
3. On a special case of the system (2.1) 368

References 371

S. Kupin and P. Yuditskii

Analogs of Nehari and Sarason theorems for character-automorphic functions and some related questions 373

0. Introduction 373
1. Groups of Widom-Carleson type and Hardy spaces of character-automorphic functions 374
2. Character-automorphic analogs of the Nehari and Sarason Theorems 378
3. A solvability criterion for the character-automorphic Nevanlinna-Pick problem and the uniqueness of solution 381
4. A solvability criterion for the Nevanlinna-Pick problem in the plane with a homogeneous compact removed 385

References 390

M.S. Livšic

The Blaschke-Potapov factorization theorem and the theory of nonselfadjoint operators 391

0. Preface 391
1. Characteristic functions 392
2. Potapov's factorization theorem 395

References 396

I.V. Mikhailova

Weyl matrix circles as a tool for uniqueness in the theory of multiplicative representation of J-inner functions 397

References 416

I.V. Mikhailova and V.P. Potapov

On a criterion of positive definiteness 419

0. Introduction 419
1. The triad 421
2. The value $\Delta(\xi)$ 423
3. The fundamental antilinear forms $B_\xi(f), L_\xi(f), C_\xi(f)$ 427
4. The matrix function $W(z,\xi)$ and the related canonical system 431
5. The relationship between the matrix function $W(z,\xi)$ and the original problem 437
6. The formulation of the Main Theorem 446
7. The discussion of the Main Theorem 447
8. An example 449

References 450

E. Russakovskii

Matrix boundary value problems with eigenvalue dependent boundary conditions (The linear case) 453

0. Introduction 453
1. Notation and remarks 454
2. The operator $\mathcal{T}$ in the space $\mathcal{L}^2_d(0,1)$ 455
3. The generalized Lagrange identity for the operator $\mathcal{T}$ 456
4. The appropriate indefinite scalar product in the Pontryagin space $\mathcal{L}^2_d(0,1)$ 457
5. Properties of the operator $\mathcal{T}$ 457
6. Comparison with earlier results 458

References 460

L.A. Sakhnovich

Weyl-Titchmarsh functions of the canonical periodical system of differential equations 463

0. Introduction 463
1. On multipliers 464
2. Weyl-Titchmarsh functions 466
3. Singular points of the Weyl matrix function 469
4. Transformation of the Weyl-Titchmarsh function 477

References 478

V.I. Smirnov

On boundary values of functions regular in a disk 481

References 494