

Contents

Preface	ix
Introduction	1
David Gieseker, Geometric Invariant Theory and the Moduli of Bundles	5
Lecture 1. Geometric Invariant Theory	7
Lecture 2. The Numerical Criterion	11
Lecture 3. The Moduli of Stable Bundles	13
References	21
Jun Li, Anti-Self-Dual Connections and Stable Vector Bundles	23
Introduction	25
Lecture 1. Hermitian Bundles, Hermitian Connections and Their Curvatures	27
Lecture 2. Hermitian-Einstein Connections and Stable Vector Bundles	33
Lecture 3. The Existence of Hermitian-Einstein Metrics	41
References	49
John W. Morgan, An Introduction to Gauge Theory	51
Lecture 1. The Context of Gauge Theory	53
1.1. Problems	55
Lecture 2. Principal Bundles and Connections	59
2.1. Principal bundles	59
2.2. Examples of principal bundles	60
2.3. The transition functions	61
2.4. Pullback bundles	62
2.5. Associated bundles	62
2.6. Universal bundles	63
2.7. Connections on smooth principal bundles	65
2.8. The differential form description of a connection	65

2.9. Existence of connections	66
2.10. Covariant differentiation	67
2.11. Problems	68
Lecture 3. Curvature and Characteristic Classes	71
3.1. The curvature of a connection as an obstruction to integrating the horizontal distribution	71
3.2. Interpretation of the curvature in terms of the connection-one form	73
3.3. The relationship of curvature and covariant differentiation	74
3.4. Characteristic classes	75
3.5. The holonomy of a connection	76
3.6. Problems	77
Lecture 4. The Space of Connections	81
4.1. The group of bundle isomorphisms of P	81
4.2. Infinite dimensional manifolds, a first version	82
4.3. The action of the group of gauge transformations on the space of connections	83
4.4. The space of gauge equivalence classes of connections	85
4.5. The local structure of the quotient space	90
4.6. Problems	90
Lecture 5. The ASD Equations and the Moduli Space	95
5.1. The ASD equations and the moduli space	95
5.2. The local structure of the moduli space	96
5.3. The generic metrics theorem	99
5.4. Reducible connections	100
5.5. Orientability of $\mathcal{M}^*(P)$	101
5.6. Variation of the metric	102
5.7. Problems	103
Lecture 6. Compactness and Gluing Theorems	109
6.1. Uhlenbeck compactness	109
6.2. Gluing together connections	112
6.3. Taubes' gluing theorem	113
6.4. The moduli space of instantons over S^4	115
6.5. The ends of $\mathcal{M}(P)$	116
6.6. Negative definite 4-manifolds	117
6.7. Problems	120
Lecture 7. The Donaldson Polynomial Invariants	123
7.1. The formalism of the Donaldson polynomial invariants	123
7.2. The μ -map	124
7.3. The Uhlenbeck compactification of the moduli space	126
7.4. Extension of the μ -map over the compactification	127
7.5. Definition of the Donaldson polynomial invariants in the stable range	128
7.6. A blow-up formula and the unstable range	129
7.7. The Donaldson series	130
7.8. Problems	131
Lecture 8. The Connected Sum Theorem	135

8.1. The main results	135
8.2. The divisors in $\mathcal{B}_2^*(Q)$ representing $\mu(x)$	136
8.3. The Taubes gluing setup of the connected sum theorem	137
8.4. The sketch of the argument for the connected sum theorem	139
8.5. The non-vanishing theorem	140
8.6. Problems	141
References	143
Ronald J. Stern, Computing Donaldson Invariants	145
Abstract	147
Lecture 1. Overview	149
1.1. Classical invariants	149
1.2. Existence	151
1.3. Uniqueness (The Donaldson Invariant)	153
Lecture 2. -2 Spheres and the Blowup Formula	161
2.1. Ruberman's theorem	161
2.2. The recursion scheme	164
2.3. Universal relations	166
2.4. The ODE	167
2.5. Solving the ODE	168
2.6. The simple-type condition	169
Lecture 3. Simple-Type Criteria and Elliptic Surfaces	173
3.1. Manifolds with big diffeomorphism group	173
3.2. A simple-type criteria	175
3.3. The Donaldson series for the $K3$ surface	176
3.4. Another simple-type criteria	176
3.5. The Donaldson series for elliptic surfaces	176
Lecture 4. Elementary Rational Blowdowns	179
4.1. Elementary rational blowdowns	179
4.2. Logarithmic transform as rational blowdown	180
4.3. The basic computational theorem	181
4.4. The Donaldson series for $E(n; p, q)$	182
Lecture 5. Taut Configurations and Horikawa Surfaces	187
5.1. Taut configurations	187
5.2. Horikawa surfaces	188
References	191
Clifford H. Taubes and James A. Bryan, Donaldson-Floer Theory	195
Abstract	197
Lecture 1. Introduction	199
1.1. Motivation	199
1.2. The moduli space and the invariants	200
1.3. A strategy to compute the invariants	202

Lecture 2. Quantization	205
2.1. Problems	205
2.2. Quantization and the Chern-Simons functional	207
Lecture 3. Simplicial Decomposition of \mathcal{M}_X^0	211
3.1. The decomposition	211
3.2. Formal consequences	213
3.3. Problems revisited	214
Lecture 4. Half-Infinite Dimensional Spaces	217
References	221