Contents

1 Introduction 1

1.1 Territory Design 1

1.2 Facility Location 5

1.3 Combining Territory Design and Facility Location 7

1.4 Overview 9

Part I Territory Design 11

2 Territory Design 13

2.1 Applications 13

2.1.1 Political Districting 13

2.1.2 Sales and Service Territory Design 15

2.1.3 Other Applications 20

2.2 A Basic Model for Territory Design 23

2.3 Overview of Solution Techniques 27

2.4 Extensions and Planning Scenarios 32

2.4.1 Extensions 33

2.4.2 Planning Scenarios 34

3 Solving the Territory Design Problem 37

3.1 General Definitions 38

3.2 Line Partitions 40

3.2.1 Definition of a Line Partition 40

3.2.2 Balance of a Line Partition 44

3.2.3 Contiguity of a Line Partition 53

3.2.4 Compactness of a Line Partition 53

3.3 The Successive Dichotomies Heuristic 54

3.3.1 Partitioning a Problem 55

3.3.2 Exploring the Partition Tree 57

3.4 Computational Results 60

3.5 Extensions and Planning Scenarios 69

3.5.1 Extensions 69

3.5.2 Planning Scenarios 73

3.6 Integration into GIS 77

Part II The Ordered Median Problem on Networks 79

4 The Ordered Median Problem 81
4.1 Introduction . . . 81
4.2 Definitions and Notations . . . 83
4.2.1 General Definitions and Notations . . . 83
4.2.2 Graphs and Networks . . . 84
4.2.3 Distances on Networks . . . 85
4.2.4 Classification Scheme . . . 91
4.3 Problem Formulation . . . 92
4.4 Basic properties . . . 95
4.4.1 Bounds on the Ordered Median Function . . . 95
4.4.2 Concavity and Convexity Results for Special Classes of Modeling Vectors . . . 104
4.5 Modeling Different Objective Functions . . . 107
4.6 Extensions to the Ordered Median Problem . . . 116
4.6.1 The Conditional Ordered Median Problem . . . 116
4.6.2 The Restricted Ordered Median Problem . . . 118
4.6.3 The Ordered Median Problem with Fixed Costs . . . 120
4.6.4 The Ordered Median Problem with General Functions of the Distance . . . 121

5 Finite Dominating Sets for the Single Facility OMP 125
5.1 The Ordered Median Problem on Undirected Networks . . . 126
5.1.1 The General Case . . . 126
5.1.2 Special Cases . . . 134
5.2 The Ordered Median Problem on Tree Networks . . . 137
5.3 The Ordered Median Problem on Directed Networks . . . 139
5.4 The Conditional Ordered Median Problem . . . 142
5.4.1 Special Cases . . . 150
5.4.2 Tree Networks . . . 150
5.5 The Restricted Ordered Median Problem . . . 152
5.5.1 Special Cases . . . 154
5.5.2 Tree Networks . . . 155
5.6 The Ordered Median Problem with Fixed Costs . . . 156
5.7 The Ordered Median Problem with General Distance Functions . . . 158

6 Algorithms for the Single Facility Ordered Median Problem 165
6.1 The Node-Restricted Ordered Median Problem . . . 165
6.2 The Ordered Median Problem on Undirected Networks . . . 167
6.2.1 Special Cases . . . 172

6.2.2 Bounds on the Ordered Median Function 173
6.3 The Ordered Median Problem on Tree Networks 177
6.3.1 The Unweighted Problem 178
6.3.2 Node-Restricted and Concave Problems 181
6.3.3 The Convex Problem 182
6.4 The Ordered Median Problem on Directed Networks 186
6.5 The Conditional Ordered Median Problem 187
6.5.1 Bounds on the Ordered Median Function 189
6.5.2 Special Cases 190
6.5.3 Tree Networks 190
6.6 The Restricted Ordered Median Problem 191
6.7 The Ordered Median Problem with Fixed Costs 192
6.8 The Ordered Median Problem with General Distance Functions 192

7 Finite Dominating Sets for the Multifacility OMP 195
7.1 The Ordered Median Problem on Undirected Networks 196
7.1.1 FDS for Special Classes of Modeling Vectors and Nonnegative Weights 199
7.1.2 FDS for Real-Valued Node and Modeling Weights 212
7.2 The Ordered p-Median Problem on Directed Networks 230
7.3 The Conditional Ordered p-Median Problem 233
7.3.1 FDS for Special Classes of Modeling Vectors and Nonnegative Weights 235
7.3.2 FDS for Real-Valued Node and Modeling Weights 240

8 Algorithms for the Multifacility Ordered Median Problem 245
8.1 The Node-Restricted Multifacility Ordered Median Problem 246
8.2 The Multifacility Ordered Median Problem on Undirected Networks . . 246
8.2.1 Algorithms for Special Classes of Modeling Vectors and Nonnegative Weights 246
8.2.2 Algorithms for Real-Valued Node and Modeling Weights 249
8.3 The Multifacility Ordered Median Problem on Tree Networks 250
8.3.1 Dynamic Programming Approach 251
8.3.2 An Alternative Approach 266
8.4 The Multifacility Ordered Median Problem on Directed Networks . . . 267
8.5 The Conditional Ordered p-Median Problem 268

9 Conclusions and Outlook 269

Symbols and Notations 271

List of Figures 275

List of Tables 277

List of Algorithms 279

Bibliography 281

Index 294