

# Contents

Preface	1
Chapter 1. Wightman axioms and vertex algebras	3
1.1. Wightman axioms of a QFT	3
1.2. $d = 2$ QFT and chiral algebras	6
1.3. Definition of a vertex algebra	11
1.4. Holomorphic vertex algebras	13
Chapter 2. Calculus of formal distributions	15
2.1. Formal delta-function	15
2.2. An expansion of a formal distribution $a(z, w)$	17
2.3. Locality	18
2.4. Taylor's formula	24
2.5. Current algebras	25
2.6. Conformal weight and the Virasoro algebra	28
2.7. Lie superalgebras of formal distributions and conformal superalgebras	31
Chapter 3. Local fields	37
3.1. Normally ordered product	37
3.2. Dong's lemma	40
3.3. Wick's theorem and a "non-commutative" generalization	42
3.4. Restricted and field representations of Lie superalgebras of formal distributions	45
3.5. Free (super)bosons	47
3.6. Free (super)fermions	52
Chapter 4. Structure theory of vertex algebras	57
4.1. Consequences of translation covariance	57

4.2. Quasisymmetry	58
4.3. Subalgebras, ideals, and tensor products	59
4.4. Uniqueness theorem	61
4.5. Existence theorem	62
4.6. Borcherds OPE formula	64
4.7. Vertex algebras associated to Lie superalgebras of formal distributions	68
4.8. Borcherds identity	71
4.9. Graded and Möbius conformal vertex algebras	72
4.10. Conformal vertex algebras	78
4.11. Field algebras	82
 Chapter 5. Examples of vertex algebras and their applications	 87
5.1. Charged free fermions	87
5.2. Boson-fermion correspondence and KP hierarchy	90
5.3. $\widehat{gl}_\infty$ and $W_{1+\infty}$	97
5.4. Lattice vertex algebras	101
5.5. Simple lattice vertex algebras	104
5.6. Root lattice vertex algebras and affine vertex algebras	111
5.7. Conformal structure for affine vertex algebras	114
5.8. Superconformal vertex algebras	121
5.9. On classification of conformal superalgebras	129
 Bibliography	 135
 Index	 139