

# Contents

Introduction	ix
Chapter 1. The Iwahori–Hecke algebra of the symmetric group	1
1. The Symmetric group	1
2. The Iwahori–Hecke algebra	5
3. The 0–Hecke algebra	10
Chapter 2. Cellular algebras	15
1. Cellular bases	15
2. Simple modules in a cellular algebra	19
Chapter 3. The modular representation theory of $\mathcal{H}$	27
1. The combinatorics of tableaux	27
2. The Murphy basis	32
3. Specht modules and Jucys–Murphy elements	39
4. Irreducible $\mathcal{H}$ –modules	45
Chapter 4. The $q$ –Schur algebra	55
1. Semistandard tableaux	55
2. A Specht filtration of $M^\mu$	59
3. The semistandard basis theorem	61
Chapter 5. The Jantzen sum formula and the blocks of $\mathcal{H}$	69
1. Gram determinants of Weyl modules	69
2. The Jantzen sum formula	81
3. The blocks of $\mathcal{S}(n)$ and $\mathcal{H}$	84
4. Irreducible Weyl modules and Specht modules	87
Chapter 6. Branching rules, canonical bases and decomposition matrices	95
1. The LLT algorithm	95
2. Decomposition maps and adjustment matrices	115
3. The Kleshchev–Brundan modular branching rules	118
4. Rules for computing decomposition matrices	122
5. The $q$ –Schur algebras and $GL_n(q)$	129
6. The Ariki–Koike algebras and cyclotomic $q$ –Schur algebras	131
Appendix A. Finite dimensional algebras over a field	137
1. Filtrations and composition series	137
2. Idempotents and indecomposable modules	138
3. The blocks of $A$	144
4. Semisimple symmetric algebras	146

Appendix B. Decomposition matrices	149
1. Decomposition matrices when $e = 2$	149
2. Decomposition matrices when $e = 3$	156
3. Decomposition matrices when $e = 4$	161
Appendix C. Elementary divisors of integral Specht modules	165
Index of notation	177
References	181
Index	187