

Contents

Introduction	1
Chapter 1. Polytopes	7
1.1. Definitions and main constructions	7
1.2. Face vectors and Dehn–Sommerville equations	11
1.3. The g -theorem	15
1.4. Upper Bound and Lower Bound theorems	18
1.5. Stanley–Reisner face rings of simple polytopes	20
Chapter 2. Topology and combinatorics of simplicial complexes	21
2.1. Abstract simplicial complexes and polyhedrons	21
2.2. Basic PL topology, and operations with simplicial complexes	23
2.3. Simplicial spheres	28
2.4. Triangulated manifolds	29
2.5. Bistellar moves	31
Chapter 3. Commutative and homological algebra of simplicial complexes	35
3.1. Stanley–Reisner face rings of simplicial complexes	35
3.2. Cohen–Macaulay rings and complexes	38
3.3. Homological algebra background	40
3.4. Homological properties of face rings: Tor-algebras and Betti numbers	42
3.5. Gorenstein complexes and Dehn–Sommerville equations	46
Chapter 4. Cubical complexes	49
4.1. Definitions and cubical maps	49
4.2. Cubical subdivisions of simple polytopes and simplicial complexes	50
Chapter 5. Toric and quasitoric manifolds	57
5.1. Toric varieties	57
5.2. Quasitoric manifolds	63
5.3. Stably complex structures, and quasitoric representatives in cobordism classes	69
5.4. Combinatorial formulae for Hirzebruch genera of quasitoric manifolds	74
5.5. Classification problems	82
Chapter 6. Moment-angle complexes	85
6.1. Moment-angle manifolds \mathcal{Z}_P defined by simple polytopes	85
6.2. General moment-angle complexes \mathcal{Z}_K	87
6.3. Cell decompositions of moment-angle complexes	89
6.4. Moment-angle complexes corresponding to joins, connected sums and bistellar moves	92

6.5.	Borel constructions and Davis–Januszkiewicz space	94
6.6.	Walk around the construction of \mathcal{Z}_K : generalizations, analogues and additional comments	97
Chapter 7.	Cohomology of moment-angle complexes and combinatorics of triangulated manifolds	101
7.1.	The Eilenberg–Moore spectral sequence	101
7.2.	Cohomology algebra of \mathcal{Z}_K	102
7.3.	Bigraded Betti numbers of \mathcal{Z}_K : the case of general K	106
7.4.	Bigraded Betti numbers of \mathcal{Z}_K : the case of spherical K	110
7.5.	Partial quotients of \mathcal{Z}_P	113
7.6.	Bigraded Poincaré duality and Dehn–Sommerville equations	117
Chapter 8.	Cohomology rings of subspace arrangement complements	125
8.1.	General arrangements and their complements	125
8.2.	Coordinate subspace arrangements and the cohomology of \mathcal{Z}_K .	127
8.3.	Diagonal subspace arrangements and the cohomology of $\Omega\mathcal{Z}_K$.	133
Bibliography		135
Index		141