

# Table of Contents

0. Introduction	1
0.1. Theorems and constructions and ergodic theory	1
0.2. A little history	2
0.3. The story and purpose of these notes	3
0.4. Acknowledgements	4
<b>Part I. Approximation and Genericity in Ergodic Theory</b>	<b>5</b>
1. Periodic processes	5
2. Genericity of approximation	10
3. Various types of approximation	12
3.1. Cyclic approximation	12
3.2. Approximation of type $(n, n + 1)$	15
3.3. $\alpha$ -weak mixing and singularity of convolutions	15
4. Spectral multiplicity of ergodic transformations	18
4.1. Essential value of spectral multiplicity	19
4.2. Transformations with arbitrary maximal spectral multiplicity	19
4.3. Cartesian powers and multiplicities bounded from below	22
4.4. Some recent results	26
5. Approximation and coding	27
6. Invariant measures for transformation with specification	37
7. Generic induced maps	42
8. Combinatorial approximation by conjugation construction	44
8.1. Introduction	44
8.2. General framework	45
8.3. Ergodicity and rotation factors	48
8.4. Non-standard transformations	50
<b>Part II. Cocycles, Cohomology and Combinatorial Constructions</b>	<b>53</b>
9. Definitions and principal constructions	54
9.1. Cocycles, coboundaries and Mackey range	55
9.2. Lipschitz cocycles, pseudo-isometries and the Ambrose–Kakutani theorem	58

9.3. Cohomological equations for measure-preserving transformations and flows	59
10. Structure of equivalence classes	62
10.1. Majorization and density in $L^1$	62
10.2. Continuous and almost differentiable representations	66
11. Rigidity and stability	68
11.1. Definitions	68
11.2. Translations of the torus and smooth rigidity	70
11.3. Stability of Hölder cocycles for transformations with specification	74
11.4. Livshitz theory	78
11.5. Invariant distributions and stability of partially hyperbolic systems	81
11.6. Stability determined by invariant distributions in parabolic systems	85
12. Wild cochains with tame coboundaries	89
12.1. Continuous cocycles over measure-preserving homeomorphisms	90
12.2. Fast approximation and $C^\infty$ cocycles	94
12.3. Minimal nonergodic diffeomorphisms of $\mathbb{T}^2$	97
12.4. Minimal nonergodic interval exchange transformations	98
13. Non-trivial cocycles	102
13.1. Two general criteria	102
13.2. The case of fast $C^\infty$ approximation	105
13.3. Weakly mixing flows on $\mathbb{T}^2$	107
13.4. Ergodicity of analytic cylindrical cascades	112
13.5. Weak mixing of special flows over interval exchange transformations	114
References	117