# Table of Contents

Introduction . . . . . 1

**Part A. The Basic Theory** . . . . . 5

*Chapter I.* Admissible Set Theory . . . . . 7

1. The Role of Urelements . . . . . 7
2. The Axioms of KPU . . . . . 9
3. Elementary Parts of Set Theory in KPU . . . . . 11
4. Some Derivable Forms of Separation and Replacement . . . . . 14
5. Adding Defined Symbols to KPU . . . . . 18
6. Definition by $\Sigma$ Recursion . . . . . 24
7. The Collapsing Lemma . . . . . 30
8. Persistent and Absolute Predicates . . . . . 33
9. Additional Axioms . . . . . 38

*Chapter II.* Some Admissible Sets . . . . . 42

1. The Definition of Admissible Set and Admissible Ordinal . . . . . 42
2. Hereditarily Finite Sets . . . . . 46
3. Sets of Hereditary Cardinality Less Than a Cardinal $\kappa$ . . . . . 52
4. Inner Models: the Method of Interpretations . . . . . 54
5. Constructible Sets with Urelements; $\mathbb{HYP}_{\mathfrak{M}}$ Defined . . . . . 57
6. Operations for Generating the Constructible Sets . . . . . 62
7. First Order Definability and Substitutable Functions . . . . . 69
8. The Truncation Lemma . . . . . 72
9. The Lévy Absoluteness Principle . . . . . 76

*Chapter III.* Countable Fragments of $\mathsf{L}_{\varkappa\omega}$ . . . . . 78

1. Formalizing Syntax and Semantics in KPU . . . . . 78
2. Consistency Properties . . . . . 84
3. $\mathfrak{M}$-Logic and the Omitting Types Theorem . . . . . 87
4. A Weak Completeness Theorem for Countable Fragments . . . . . 92
5. Completeness and Compactness for Countable Admissible Fragments 95

6. The Interpolation Theorem . . . . . 103
7. Definable Well-Orderings . . . . . 105
8. Another Look at Consistency Properties . . . . . 109

*Chapter IV.* Elementary Results on $\mathbb{HYP}_{\mathfrak{M}}$ . . . . . 113

1. On Set Existence . . . . . 113
2. Defining $\Pi^1_1$ and $\Sigma^1_1$ Predicates . . . . . 116
3. $\Pi^1_1$ and $\Delta^1_1$ on Countable Structures . . . . . 122
4. Perfect Set Results . . . . . 127
5. Recursively Saturated Structures . . . . . 137
6. Countable $\mathfrak{M}$-Admissible Ordinals . . . . . 144
7. Representability in $\mathfrak{M}$-Logic . . . . . 146

**Part B. The Absolute Theory** . . . . . 151

*Chapter V.* The Recursion Theory of $\Sigma_1$ Predicates on Admissible Sets . . 153
1. Satisfaction and Parametrization . . . . . 153
2. The Second Recursion Theorem for KPU . . . . . 156
3. Recursion Along Well-founded Relations . . . . . 158
4. Recursively Listed Admissible Sets . . . . . 164
5. Notation Systems and Projections of Recursion Theory . . . . . 168
6. Ordinal Recursion Theory: Projectible and Recursively Inaccessible Ordinals . . . . . 173
7. Ordinal Recursion Theory: Stability . . . . . 177
8. Shoenfield's Absoluteness Lemma and the First Stable Ordinal . . 189

*Chapter VI.* Inductive Definitions . . . . . 197

1. Inductive Definitions as Monotonic Operators . . . . . 197
2. $\Sigma$ Inductive Definitions on Admissible Sets . . . . . 205
3. First Order Positive Inductive Definitions and $\mathbb{HYP}_{\mathfrak{M}}$ . . . . . 211
4. Coding $\mathbb{HF}_{\mathfrak{M}}$ on $\mathfrak{M}$ . . . . . 220
5. Inductive Relations on Structures with Pairing . . . . . 230
6. Recursive Open Games . . . . . 242

**Part C. Towards a General Theory** . . . . . 255

*Chapter VII.* More about $L_{\infty\omega}$ . . . . . 257

1. Some Definitions and Examples . . . . . 257
2. A Weak Completeness Theorem for Arbitrary Fragments . . . . . 262
3. Pinning Down Ordinals: the General Case . . . . . 270
4. Indiscernibles and upward Löwenheim-Skolem Theorems . . . . . 276
5. Partially Isomorphic Structures . . . . . 292
6. Scott Sentences and their Approximations . . . . . 297
7. Scott Sentences and Admissible Sets . . . . . 303

*Chapter VIII.* Strict $\Pi^1_1$ Predicates and König Principles . . . . . 311

1. The König Infinity Lemma . . . . . 311
2. Strict $\Pi^1_1$ predicates: Preliminaries . . . . . 315
3. König Principles on Countable Admissible Sets . . . . . 321
4. König Principles $K_1$ and $K_2$ on Arbitrary Admissible Sets . . . . . 326
5. König's Lemma and Nerode's Theorem: a Digression . . . . . 334
6. Implicit Ordinals on Arbitrary Admissible Sets . . . . . 339
7. Trees and $\Sigma_1$ Compact Sets of Cofinality $\omega$ . . . . . 343
8. $\Sigma_1$ Compact Sets of Cofinality Greater than $\omega$ . . . . . 352
9. Weakly Compact Cardinals . . . . . 356

*Appendix.* Nonstandard Compactness Arguments and the Admissible Cover . 365

1. Compactness Arguments over Standard Models of Set Theory . . . 365
2. The Admissible Cover and its Properties . . . . . 366
3. An Interpretation of KPU in KP . . . . . 372
4. Compactness Arguments over Nonstandard Models of Set Theory . 378

References . . . . . 380

Index of Notation . . . . . 386

Subject Index . . . . . 388