

TABLE OF CONTENTS

	Page
	vii
PREFACE.....	1
CHAPTER I: FORMAL MATHEMATICAL SYSTEMS.....	1
#A. Elementary Formal Systems.....	2
§0. Motivation.....	2
§1. Definition of an Elementary Formal System.....	3
§2. Alternative Formulation of Elementary Formal Systems.....	5
§3. Representability.....	6
§4. Mathematical Systems.....	9
#B. Recursive Enumerability.....	10
§5. Recursively Enumerable Attributes of Positive Integers.....	10
§6. Gödel Numbering.....	11
§7. The Universal System U.....	12
§8. The Recursive Unsolvability of U.....	14
Appendix.....	16
CHAPTER II: FORMAL REPRESENTABILITY AND RECURSIVE ENUMERABILITY.....	19
§0. Some Preliminary Principles.....	19
#A. Closure Properties.....	20
§1. Closure Under Existential Definability.....	21
§2. Solvability over K.....	26
#B. Recursive Enumerability.....	28
§3. Recursive Enumerability of Some Basic Arithmetical Attributes.....	28
§4. Recursive and Partial Recursive Functions.....	29
§5. Finite Quantification; Constructive Definability	29
#C. Transformations on Alphabets; Gödel Numbering.....	31
§6. Extension of Alphabets.....	31
§7. Dyadic Gödel Numbering.....	32
§8. Solvability.....	33
§9. Lexicographical Ordering; n-adic Representation of Numbers.....	34
§10. Admissible Gödel Correspondences.....	37
§11. Further Facts About Admissibility [Optional].....	37
#D. A Brief Summary.....	38
CHAPTER III: INCOMPLETENESS AND UNDECIDABILITY.....	39
#A. Incompleteness.....	41
§1. Representation Systems.....	41
§2. First Diagonalization Lemma; Tarski's Theorem.....	42
§3. Consistency and Completeness; Gödel's Theorem.....	45
§4. Complete Representability and Definability in Z.....	46
§5. Separability within Z; Rosser's Theorems.....	47
§6. Symmetric Systems.....	49
§7. Extensions.....	50
#B. Undecidability.....	51
§8. Systems With an "Effective" Representation Function.....	52

CONTENTS

Page

§9.	Undecidability.....	53
§10.	Normality.....	55
§11.	Additional Theorems.....	55
§12.	Universal Systems.....	56
§13.	Undecidability and Incompleteness.....	56
#C.	Undecidability and Recursive Inseparability.....	58
§14.	Definability in Formal Systems.....	58
§15.	Extensions.....	58
§16.	Recursive Inseparability.....	58
§17.	Separation of R. I Sets Within Systems.....	59
§18.	Rosser Systems.....	60
§19.	Recursive Inseparability of the Diagonal Sets T^* , R^*	61
CHAPTER IV:	RECURSIVE FUNCTION THEORY.....	65
#A.	Effective Operations and Fixed Point Theorems.....	65
§1.	Enumeration Theorem.....	65
§2.	Indexing.....	67
§3.	Iteration Theorems for r.e. Relations.....	67
§4.	Effective Operations.....	70
§5.	Fixed Point Theorems.....	72
§6.	Double Recursion Theorems.....	75
#B.	Constructive Arithmetic and Rudimentary Attributes..	77
§7.	Some Preliminaries.....	78
§8.	Dyadic Concatenation.....	79
§9.	Rudimentary Attributes.....	81
§10.	Pure Elementary Formal Systems.....	84
§11.	Arithmetization of Elementary Dyadic Arithmetics.....	85
#C.	Enumeration and Normal Form Theorems.....	89
§12.	Kleene Enumeration Theorem.....	89
§13.	Separation of Differences of r.e. Sets.....	90
§14.	Partial Recursive Functions.....	90
§15.	Functional Indexing.....	91
CHAPTER V:	CREATIVITY AND EFFECTIVE INSEPARABILITY.....	93
#A.	Creativity and Effective Inseparability.....	93
§1.	Productive and Creative Sets; Recursive and Effective Inseparability.....	93
§2.	Many-one and One-one Reducibility.....	95
§3.	Creative Systems.....	97
§4.	Effective Inseparability.....	98
§5.	Effective Rosser Systems.....	99
#B.	Further Theory of Productive Sets.....	100
§6.	Weakly Productive Functions.....	100
§7.	Uniform Reducibility.....	102
§8.	Universal Sets.....	104
§9.	On Uniform Reducibility.....	104
§10.	Uniform Representability.....	106
§11.	Bi-Uniformity.....	106
#C.	Effective Inseparability and Double Productivity....	107
§12.	Doubly Productive Pairs.....	107
§13.	Reducibility of Pairs to Pairs.....	111
#D.	Double Universality.....	113
§14.	Doubly Universal and Totally Doubly Universal Pairs.....	113
§15.	Uniform Reducibility.....	113

§16. Weakly Doubly Productive Pairs.....	114
§17. On Doubly Productive Pairs.....	115
§18. Application to Rosser Systems.....	118
§19. Uniform Reducibility.....	118
§20. A Generalization of Effective Inseparability.....	120
 #E. Double Isomorphisms.....	122
§21. Double Isomorphism.....	122
§22. 1 - 1 Equivalence and Double Isomorphism.....	124
§23. Double Isomorphism of Rosser Systems.....	125
 SUPPLEMENT.....	127
§1. Theories.....	127
§2. Calculability of Functions; Normality.....	130
§3. ω -Consistency; Enumerability Within (T); Gödel's Theorem.....	131
§4. Rosser's Construction.....	132
§5. Gödel Theories and Rosser Theories.....	134
§6. Theories in Which Plus and Times are Definable.....	136
§7. Some Special Theories.....	137
§8. Essential Undecidability.....	137
§9. Essential Creativity.....	138
§10. Exact Rosser Theories.....	139
 REFERENCE AND BRIEF BIBLIOGRAPHY.....	141
INDEX OF DEFINITIONS AND NOTATIONS.....	143