# CONTENTS

*Editors' Preface* *page* ix

*Acknowledgments* xii

*Author's Introduction* 1

CHAPTER 1

1 A Problem and a Conjecture 6

2 A Proof 7

3 Criticism of the Proof by Counterexamples which are Local but not Global 10

4 Criticism of the Conjecture by Global Counterexamples 13

a. Rejection of the conjecture. The method of surrender 13

b. Rejection of the counterexample. The method of monster-barring 14

c. Improving the conjecture by exception-barring methods. Piecemeal exclusions. Strategic withdrawal or playing for safety 24

d. The method of monster-adjustment 30

e. Improving the conjecture by the method of lemma-incorporation. Proof-generated theorem versus naive conjecture 33

5 Criticism of the Proof-Analysis by Counterexamples which are Global but not Local. The Problem of Rigour 42

a. Monster-barring in defence of the theorem 42

b. Hidden lemmas 43

c. The method of proof and refutations 47

d. Proof versus proof-analysis. The relativisation of the concepts of theorem and rigour in proof-analysis 50

6 Return to Criticism of the Proof by Counterexamples which are Local but not Global. The Problem of Content 57

a. Increasing content by deeper proofs 57

b. Drive towards final proofs and corresponding sufficient and necessary conditions 63

c. Different proofs yield different theorems 65

7 The Problem of Content Revisited *page* 66
a. The naiveté of the naive conjecture 66
b. Induction as the basis of the method of proofs and refutations 68
c. Deductive guessing versus naive guessing 70
d. Increasing content by deductive guessing 76
e. Logical versus heuristic counterexamples 82
8 Concept-Formation 83
a. Refutation by concept-stretching. A reappraisal of monster-barring – and of the concepts of error and refutation 83
b. Proof-generated versus naive concepts. Theoretical versus naive classification 88
c. Logical and heuristic refutations revisited 92
d. Theoretical versus naive concept-stretching. Continuous versus critical growth 93
e. The limits of the increase in content. Theoretical versus naive refutations 96
9 How Criticism may turn Mathematical Truth into Logical Truth 99
a. Unlimited concept-stretching destroys meaning and truth 99
b. Mitigated concept-stretching may turn mathematical truth into logical truth 102

CHAPTER 2
Editors' Introduction 106
1 Translation of the Conjecture into the 'Perfectly Known' Terms of Vector-Algebra. The Problem of Translation 106
2 Another Proof of the Conjecture 116
3 Some Doubts about the Finality of the Proof. Translation Procedure and the Essentialist versus the Nominalist Approach to Definitions 119

APPENDIX 1
Another Case-Study in the Method of Proofs and Refutations
1 Cauchy's Defence of the 'Principle of Continuity' 127
2 Seidel's Proof and the Proof-Generated Concept of Uniform Convergence 131
3 Abel's Exception-Barring Method 133

4 Obstacles in the Way of the Discovery of the Method of Proof-Analysis *page* 136

APPENDIX 2

The Deductivist versus the Heuristic Approach

1 The Deductivist Approach 142

2 The Heuristic Approach. Proof-Generated Concepts 144

a. Uniform convergence 144

b. Bounded variation 146

c. The Carathéodory definition of measurable set 152

*Bibliography* 155

*Index of Names* 167

*Index of Subjects* 170