

Contents

PREFACE	v
INTRODUCTION	1
00. Logic	1
01. Names	3
02. Constants and variables	9
03. Functions	15
04. Propositions and propositional functions	23
05. Improper symbols, connectives	31
06. Operators, quantifiers	39
07. The logistic method	47
08. Syntax	58
09. Semantics	64
CHAPTER I. The Propositional Calculus	69
10. The primitive basis of P_1	69
11. Definitions	74
X 12. Theorems of P_1	81
Exercises 12.	85
13. The deduction theorem	86
14. Some further theorems and metatheorems of P_1	91
Exercises 14.	93
15. Tautologies, the decision problem	94
Exercises 15.	102
16. Duality	106
17. Consistency	108
18. Completeness	109
Exercises 18.	111
19. Independence	112
Exercises 19.	115
CHAPTER II. The Propositional Calculus (Continued)	119
20. The primitive basis of P_2	119
X 21. The deduction theorem for P_2	120

22.	Some further theorems and metatheorems of P_2	121
23.	Relationship of P_2 to P_1	125
	Exercises 23.	128
24.	Primitive connectives for the propositional calculus	129
	Exercises 24.	134
25.	Other formulations of the propositional calculus	136
	Exercises 25.	138
26.	Partial systems of propositional calculus	140
	Exercises 26.	143
27.	Formulations employing axiom schemata.	148
28.	Extended propositional calculus and protothetic	151
	Exercises 28.	154
29.	Historical notes	155
	Exercises 29.	166
CHAPTER III. Functional Calculi of First Order		168
30.	The primitive basis of F^1	169
	Exercises 30.	176
31.	Propositional calculus	178
32.	Consistency of F^1	180
33.	Some theorem schemata of F^1	186
34.	Substitutivity of equivalence	189
	Exercises 34	191
35.	Derived rules of substitution	191
	Exercises 35	195
36.	The deduction theorem.	196
37.	Duality	201
38.	Some further theorem schemata.	205
	Exercises 38.	206
39.	Prenex normal form	209
	Exercises 39.	212
CHAPTER IV. The Pure Functional Calculus of First Order		218
40.	An alternative formulation	218
	Exercises 40.	220
41.	Independence	220
	Exercises 41.	223
42.	Skolem normal form	224
43.	Validity and satisfiability.	227
	Exercises 43.	231

44. Gödel's completeness theorem	233
45. Löwenheim's theorem and Skolem's generalization	238
Exercises 45.	245
46. The decision problem, solution in special cases	246
Exercises 46.	257
47. Reductions of the decision problem	270
Exercises 47.	280
48. Functional calculus of first order with equality.	280
Exercises 48.	282
49. Historical notes	288
CHAPTER V. Functional Calculi of Second Order	295
50. The primitive basis of F_2^2	295
51. Propositional calculus and laws of quantifiers	297
52. Equality	300
Exercises 52.	302
53. Consistency of F_2^2	306
54. Henkin's completeness theorem	307
Exercises 54.	315
55. Postulate theory	317
Exercises 55.	333
56. Well-ordering of the individuals	341
Exercises 56.	342
57. Axiom of infinity	342
Exercises 57.	345
58. The predicative and ramified functional calculi of second order	346
Exercises 58.	354
59. Axioms of reducibility	354
Exercises 59.	356
INDEX OF DEFINITIONS	357
INDEX OF AUTHORS CITED	373
ERRATA	377