

# CONTENTS OF VOLUME I

	PAGE
PREFACE . . . . .	v
ALPHABETICAL LIST OF PROPOSITIONS REFERRED TO BY NAMES . . . . .	xii
INTRODUCTION TO THE SECOND EDITION . . . . .	xiii
INTRODUCTION . . . . .	1
CHAPTER I. PRELIMINARY EXPLANATIONS OF IDEAS AND NOTATIONS . . . . .	4
CHAPTER II. THE THEORY OF LOGICAL TYPES . . . . .	37
CHAPTER III. INCOMPLETE SYMBOLS . . . . .	66
PART I. MATHEMATICAL LOGIC	
Summary of Part I . . . . .	87
SECTION A. THE THEORY OF DEDUCTION . . . . .	90
*1. Primitive Ideas and Propositions . . . . .	91
*2. Immediate Consequences of the Primitive Propositions . . . . .	98
*3. The Logical Product of two Propositions . . . . .	109
*4. Equivalence and Formal Rules . . . . .	115
*5. Miscellaneous Propositions . . . . .	123
SECTION B. THEORY OF APPARENT VARIABLES . . . . .	127
*9. Extension of the Theory of Deduction from Lower to Higher Types of Propositions . . . . .	127
*10. Theory of Propositions containing one Apparent Variable . . . . .	138
*11. Theory of two Apparent Variables . . . . .	151
*12. The Hierarchy of Types and the Axiom of Reducibility . . . . .	161
*13. Identity . . . . .	168
*14. Descriptions . . . . .	173
SECTION C. CLASSES AND RELATIONS . . . . .	187
*20. General Theory of Classes . . . . .	187
*21. General Theory of Relations . . . . .	200
*22. Calculus of Classes . . . . .	205
*23. Calculus of Relations . . . . .	213
*24. The Universal Class, the Null Class, and the Existence of Classes . . . . .	216
*25. The Universal Relation, the Null Relation, and the Existence of Relations . . . . .	228

	PAGE
SECTION D. LOGIC OF RELATIONS . . . . .	231
*30. Descriptive Functions . . . . .	232
*31. Converses of Relations . . . . .	238
*32. Referents and Relata of a given Term with respect to a given Relation . . . . .	242
*33. Domains, Converse Domains, and Fields of Relations . . . . .	247
*34. The Relative Product of two Relations . . . . .	256
*35. Relations with Limited Domains and Converse Domains . . . . .	265
*36. Relations with Limited Fields . . . . .	277
*37. Plural Descriptive Functions . . . . .	279
*38. Relations and Classes derived from a Double Descriptive Function . . . . .	296
Note to Section D . . . . .	299
SECTION E. PRODUCTS AND SUMS OF CLASSES . . . . .	302
*40. Products and Sums of Classes of Classes . . . . .	304
*41. The Product and Sum of a Class of Relations . . . . .	315
*42. Miscellaneous Propositions . . . . .	320
*43. The Relations of a Relative Product to its Factors . . . . .	324
PART II. PROLEGOMENA TO CARDINAL ARITHMETIC	
Summary of Part II . . . . .	329
SECTION A. UNIT CLASSES AND COUPLES . . . . .	331
*50. Identity and Diversity as Relations . . . . .	333
*51. Unit Classes . . . . .	340
*52. The Cardinal Number 1 . . . . .	347
*53. Miscellaneous Propositions involving Unit Classes . . . . .	352
*54. Cardinal Couples . . . . .	359
*55. Ordinal Couples . . . . .	366
*56. The Ordinal Number 2, . . . . .	377
SECTION B. SUB-CLASSES, SUB-RELATIONS, AND RELATIVE TYPES . . . . .	386
*60. The Sub-Classes of a given Class . . . . .	388
*61. The Sub-Relations of a given Relation . . . . .	393
*62. The Relation of Membership of a Class . . . . .	395
*63. Relative Types of Classes . . . . .	400
*64. Relative Types of Relations . . . . .	410
*65. On the Typical Definition of Ambiguous Symbols . . . . .	415
SECTION C. ONE-MANY, MANY-ONE, AND ONE-ONE RELATIONS . . . . .	418
*70. Relations whose Classes of Referents and of Relata belong to given Classes . . . . .	420
*71. One-Many, Many-One, and One-One Relations . . . . .	426
*72. Miscellaneous Propositions concerning One-Many, Many-One, and One-One Relations . . . . .	441
*73. Similarity of Classes . . . . .	455
*74. On One-Many and Many-One Relations with Limited Fields . . . . .	468

	PAGE
SECTION D. SELECTIONS . . . . .	478
*80. Elementary Properties of Selections . . . . .	483
*81. Selections from Many-One Relations . . . . .	496
*82. Selections from Relative Products . . . . .	501
*83. Selections from Classes of Classes . . . . .	508
*84. Classes of Mutually Exclusive Classes . . . . .	517
*85. Miscellaneous Propositions . . . . .	525
*88. Conditions for the Existence of Selections . . . . .	536
SECTION E. INDUCTIVE RELATIONS . . . . .	543
*90. On the Ancestral Relation . . . . .	549
*91. On Powers of a Relation . . . . .	558
*92. Powers of One-Many and Many-One Relations . . . . .	573
*93. Inductive Analysis of the Field of a Relation . . . . .	579
*94. On Powers of Relative Products . . . . .	588
*95. On the Equi-factor Relation . . . . .	596
*96. On the Posterity of a Term . . . . .	607
*97. Analysis of the Field of a Relation into Families . . . . .	623
APPENDIX A	
*8. The Theory of Deduction for Propositions containing Apparent Variables . . . . .	635
APPENDIX B	
*89. Mathematical Induction . . . . .	650
APPENDIX C	
Truth-Functions and others . . . . .	659
LIST OF DEFINITIONS . . . . .	667