

Contents

Keys to Russell's works and Cross references	x
Acknowledgements	xi
Preface by Ivor Grattan-Guinness	xiii
Introduction	1
1. Methodological and logicist background	5
1.1. Boole and Peirce	5
1.2. Dedekind and Cantor	13
1.3. Couturat and Whitehead	19
1.4. Bradley and Moore	27
1.5. <i>Foundations of geometry</i>	36
2. The unpublished mathematical philosophy: 1898-1900	44
2.1. The genesis of the 1898-1900 manuscripts	44
2.2. Logic, mathematics and ontology	49
2.3. The evolution of the main concepts	57
2.4. Concepts, axioms, presupposition and implication	62
2.5. The contradiction and the infinite	69
2.6. Relations and the 'principle of abstraction'	72
2.7. The method of definition	77
2.8. The gradual approach to Cantor	81
2.8.1. The first contacts and opinions	81
2.8.2. The reasons for the rejection	86

3.	The contribution of Peano and his school	91
3.1.	Logic	92
3.1.1.	Objective and stages	92
3.1.2.	Primitives, logical order and interdefinability	93
3.1.3.	Implication, inclusion and membership	97
3.1.4.	Classes, propositions and individuals	99
3.1.5.	Mathematical propositions and quantification	101
3.1.6.	Relations, functions, classes, properties and propositions	103
3.2.	Arithmetic	105
3.2.1.	The axioms and their interpretation: Dedekind	105
3.2.2.	The definability of number	107
3.2.3.	Real numbers: construction and definition	108
3.2.4.	The 'logician' arithmetic: Cantor	110
3.3.	Geometry	113
3.3.1.	The geometric calculus and the principles of geometry	113
3.3.2.	The 'logician' geometry	117
3.4.	The method	119
3.4.1.	Axiomatics	119
3.4.2.	Definitions	121
3.4.3.	The definition by abstraction	124
3.4.4.	Simplicity, analysis and intuition	125
3.5.	Peano's followers and their contributions	127
3.5.1.	The various improvements	127
3.5.2.	The transformation of definitions by abstraction into nominal ones	131
4.	The principles of mathematics	135
4.1.	The reaction to the Congress of 1900	135
4.1.1.	The notes to the manuscripts: from Moore to Peano	135
4.1.2.	The first writings	138
4.1.3.	The acceptance of Cantor	141
4.2.	Logic	144
4.2.1.	The indefinables and the propositional function	144
4.2.2.	Relations	151

4.3. Arithmetic	155
4.3.1. The definition of cardinal number	155
4.3.2. Finite and infinite	162
4.3.3. Quantity	164
4.3.4. Order	166
4.3.5. Ordinal numbers	169
4.3.6. Real numbers	173
4.4. Geometry	175
4.5. What Russell learned from Peano	181
5. Philosophical and methodological problems	185
5.1. Origin and evolution of Russell's logicism	185
5.2. The principle of abstraction	189
5.2.1. Origin and evolution	189
5.2.2. Assumptions and implications	194
5.3. The constructive definition	205
5.3.1. Nominal definitions	205
5.3.2. Mathematical and philosophical definitions	208
5.3.3. Analysis and ordinary language	211
5.4. Relational logic and ontology	215
Bibliography	224
B.1. Works by Russell	224
B.1.1. Unpublished manuscripts	224
B.1.2. Published or unpublished correspondence	225
B.1.3. Published works	225
B.2. Works by other authors	227