

# TABLE OF CONTENTS

IN MEMORY OF E. W. BETH (1908–1964)	XI
TRANSLATOR'S INTRODUCTION	XIII
FOREWORD	XXI

## EVERT W. BETH / PART ONE

PRELIMINARY	3
CHAPTER I. MATHEMATICAL REASONING CANNOT BE ANALYSED BY TRADITIONAL SYLLOGISTICS	6
1. Descartes	6
2. The Locke-Berkeley problem	7
3. Solutions of Berkeley, Hume and Kant	8
4. Analytic and synthetic judgments	12
5. The intuitionism of Descartes and Kant	14
6. Non-Euclidean geometry	17
7. Recent forms of intuitionism: F. A. Lange, L. Brunschvicg, E. Goblot, H. Poincaré, L. E. J. Brouwer	18
CHAPTER II. THE PSYCHOLOGICAL INTERPRETATION OF MATHEMATICAL REASONING	24
8. J. Stuart Mill	24
9. W. Stanley Jevons' critique	25
10. E. Mach, Th. Ziehen, G. Störing and G. Heymans	26
11. The supposed anti-psychologism of E. Husserl	30
12. F. Enriques and G. Mannoury	33

CHAPTER III. THE LOGICIST TRADITION

13. Aristotle's views: agreement with the practice of Greek mathematics	36
14. Pascal	38
15. Leibniz: demonstration of axioms	39
16. Frege: influence on Husserl and Heymans	41
17. Russell: the crisis of foundations	42
18. The Set Theorists: Cantor and Zermelo	44
19. Other reactions: the intuitionism of Brouwer, the psychologism of Mannoury and Enriques, the radical formalism of Hilbert	46
20. The Gödelian crisis	53
21. Natural deduction: Gentzen, Curry, Lorenzen	59
22. Syntax and semantics	68
23. The method of semantic tableaux	71
24. Algebraic and topological concepts	81

CHAPTER IV. STRICT DEMONSTRATION AND HEURISTIC PROCEDURES

	86
25. The typology of mathematicians	86
26. Views of Poincaré, Hadamard, Polya	87
27. Search for a method which is both heuristic and demonstrative: Descartes and the analysis of the Ancients	93
28. Leibniz and the decision problem	95
29. Persistence of more primitive levels: Archimedes' method	96
30. Original thought: creation or invention, construction or discovery? The Platonist reply: Frege, Cantor and Hermite	98

CHAPTER V. INTUITIVE STRUCTURES AND FORMALISED MATHEMATICS

	101
31. Spatial intuition: Kant, Helmholtz, F. Klein, Nicod, Whitehead and Tarski	101
32. Temporal intuition: Kant, Bergson, Brouwer and De Groot	105

TABLE OF CONTENTS

33. Finitist intuition according to Hilbert and the intuition of the infinite	108
34. Platonism as a real or illusory intuitive vision: the nominalist critique	111
<b>CHAPTER VI. "THINKING MACHINES" AND MATHEMATICAL THOUGHT</b>	<b>114</b>
35. Formalisation and the construction of a "thinking machine"	114
36. The construction of a "thinking machine" presupposes the solution of a decision problem	115
37. The irreducibility of the "leap from the end to the means" according to Brouwer	118
38. Recursive functions: unsolvable problems, absolute unsolvability	119
39. The two degrees of freedom of mathematical thought: solving a problem and setting a problem	123
40. Acquired self-evidence according to Bernays	124
<b>NOTE ON THE IDEA OF A "THINKING MACHINE"</b> by Jean-Blaise Grize	<b>127</b>
<b>JEAN PIAGET / PART TWO</b>	
<b>PRELIMINARY</b>	<b>131</b>
<b>CHAPTER VII. LESSONS OF THE HISTORY OF THE RELATIONS BETWEEN LOGIC AND PSYCHOLOGY</b>	<b>137</b>
41. The three stages of the history of the relations between logical and psychological investigations	137
42. The need for co-ordination	143
43. The genetic viewpoint and the normative viewpoint	153
<b>CHAPTER VIII. GENERAL PSYCHOLOGICAL PROBLEMS OF LOGICO-MATHEMATICAL THOUGHT</b>	<b>163</b>
<i>A. The problem of structures</i>	163
44. Broubaki's "matrix structures"	164

45. The structures of classes and relations in the subject's actions and operations. The formalisation of a "grouping"	166
46. The two forms of reversibility (inversion and reciprocity) and their final combination in a group of four transformations	175
47. The primacy of topology in the child's geometry	183
48. Relations between the three elementary structures and Bourbaki's matrix structures	186
CHAPTER IX. GENERAL PSYCHOLOGICAL PROBLEMS OF LOGICO-MATHEMATICAL THOUGHT (Continued)	191
B. <i>Self-evidence, intuition and invention</i>	191
49. Self-evidence, its variations and logical necessity	191
50. Invention and discovery	198
51. The multiple forms of mathematical "intuition"	208
CHAPTER X. THE PSYCHOLOGICAL PROBLEMS OF "PURE" THOUGHT	226
52. The genetic roots of pure mathematics	226
53. The psychological problem of pure mathematics	242
54. The psychological reasons for formalisation	247
55. How a formalisation of ordinary thought brings together the genetic and axiomatic methods	256
CHAPTER XI. SOME CONVERGENCES BETWEEN FORMAL AND GENETIC ANALYSES	259
56. The construction of natural numbers	259
57. The difficulties of logical reductionism	272
58. The limits of formalisation	276
CHAPTER XII. EPISTEMOLOGICAL PROBLEMS WITH LOGICAL AND PSYCHOLOGICAL RELEVANCE	281
59. Empiricist interpretation and apriorism	281

## TABLE OF CONTENTS

60. The nominalist or linguistic interpretation of mathematics	285
61. The Platonist interpretation of mathematics	290
62. The interpretation of mathematics by the laws of the general co-ordination of actions	296
<b>GENERAL CONCLUSIONS</b> by Evert W. Beth and Jean Piaget	305
<b>BIBLIOGRAPHY</b>	313
<b>NAME INDEX</b>	318
<b>SUBJECT INDEX</b>	321