CONTENTS

Preface	vii
0. Induction: some remarks	1
0.1. Induction over the set of natural numbers	1
0.2. Induction over other structures	5
0.3. Tree-structures and trees	12
1. Propositional languages	22
1.1. Propositional languages	22
1.2. Expressions	23
1.3. Formulas	25
1.4. Subformulas	28
1.5. Substitution	31
2. Common logics	34
2.1. Logics	34
2.2. Some common conditions on logics	36
2.3. Further conditions on logics	38
2.4. Syntactic equivalence	42
2.5. Extensions	44
2.6. The lemmas of Tukey and Lindenbaum	47
3. Boolean logics	51
3.1. Boolean operators	51
3.2. Boolean semantics	52
3.3. Types of Boolean operators	56
3.4. A completeness theorem in Boolean logic	59
3.5. Congruence in Boolean logic	63
3.6. Particular truth-value functions: the Big Seven	66
3.7. Arbitrary truth-value functions	73
4. Pre-classical logics	85
4.1. Expressibility	85
4.2. Boolean extensions	88
4.3. A conservation theorem in pre-classical logic	92
4.4. The Big Seven again	94

4.5. The Deduction Theorem and the Finiteness Theorem in pre-classical logic 98
4.6. Makinson's Warning 100

5. Classical logics 106
5.1. Replacement of tautological equivalents: formulas 106
5.2. Replacement of tautological equivalents: sets of formulas 108
5.3. Classical operators 110
5.4. The lattice of classical logics 113
5.5. The Isomorphism Theorem 115

6. Modal logics: a postscript 125
6.1. Two ambitions 125
6.2. A question in modal logic 126
6.3. Some translations between particular modal languages 130
6.4. Two theorems on translations 134
6.5. A definition of modal logic 138

References 143

Index of terms 145

Index of symbols 150

Index of names 152