# CONTENTS

Preface xix

Part I Basic Thoughts and Equations

1 **Philosophy of Computational Fluid Dynamics** 3
1.1 Computational Fluid Dynamics: Why? 4
1.2 Computational Fluid Dynamics as a Research Tool 6
1.3 Computational Fluid Dynamics as a Design Tool 9
1.4 The Impact of Computational Fluid Dynamics—Some Other Examples 13
1.4.1 Automobile and Engine Applications 14
1.4.2 Industrial Manufacturing Applications 17
1.4.3 Civil Engineering Applications 19
1.4.4 Environmental Engineering Applications 20
1.4.5 Naval Architecture Applications (Submarine Example) 22
1.5 Computational Fluid Dynamics: What Is It? 23
1.6 The Purpose of This Book 32

2 **The Governing Equations of Fluid Dynamics: Their Derivation, a Discussion of Their Physical Meaning, and a Presentation of Forms Particularly Suitable to CFD** 37
2.1 Introduction 38
2.2 Models of the Flow 40
2.2.1 Finite Control Volume 41
2.2.2 Infinitesimal Fluid Element 42
2.2.3 Some Comments 42
2.3 The Substantial Derivative (Time Rate of Change Following a Moving Fluid Element 43
2.4 The Divergence of the Velocity: Its Physical Meaning 47
2.4.1 A Comment 48

2.5 The Continuity Equation 49
2.5.1 Model of the Finite Control Volume Fixed in Space 49
2.5.2 Model of the Finite Control Volume Moving with the Fluid 51
2.5.3 Model of an Infinitesimally Small Element Fixed in Space 53
2.5.4 Model of an Infinitesimally Small Fluid Element Moving with the Flow 55
2.5.5 All the Equations Are One: Some Manipulations 56
2.5.6 Integral versus Differential Form of the Equations: An Important Comment 60
2.6 The Momentum Equation 60
2.7 The Energy Equation 66
2.8 Summary of the Governing Equations for Fluid Dynamics: With Comments 75
2.8.1 Equations for Viscous Flow (the Navier–Stokes Equations) 75
2.8.2 Equations for Inviscid Flow (the Euler Equations) 77
2.8.3 Comments on the Governing Equations 78
2.9 Physical Boundary Conditions 80
2.10 Forms of the Governing Equations Particularly Suited for CFD: Comments on the Conservation Form, Shock Fitting, and Shock Capturing 82
2.11 Summary 92
Problems 93

**3 Mathematical Behavior of Partial Differential Equations: The Impact on CFD** 95
3.1 Introduction 95
3.2 Classification of Quasi-Linear Partial Differential Equations 97
3.3 A General Method of Determining the Classification of Partial Differential Equations: The Eigenvalue Method 102
3.4 General Behavior of the Different Classes of Partial Differential Equations: Impact on Physical and Computational Fluid Dynamics 105
3.4.1 Hyperbolic Equations 106
3.4.2 Parabolic Equations 111
3.4.3 Elliptic Equations 117
3.4.4 Some Comments: The Supersonic Blunt Body Problem Revisited 119
3.5 Well-Posed Problems 120
3.6 Summary 121
Problems 121

Part II Basics of the Numerics

**4 Basic Aspects of Discretization** 125
4.1 Introduction 125
4.2 Introduction to Finite Differences 128

4.3 Difference Equations 142
4.4 Explicit and Implicit Approaches: Definitions and Contrasts 145
4.5 Errors and an Analysis of Stability 153
4.5.1 Stability Analysis: A Broader Perspective 165
4.6 Summary 165

*GUIDEPOST* 166

Problems 167

**5 Grids with Appropriate Transformations** 168
5.1 Introduction 168
5.2 General Transformation of the Equations 171
5.2 Metrics and Jacobians 178
5.4 Form of the Governing Equations Particularly Suited for CFD Revisited: The Transformed Version 183
5.5 A Comment 186
5.6 Stretched (Compressed) Grids 186
5.7 Boundary-Fitted Coordinate Systems; Elliptic Grid Generation 192

*GUIDEPOST* 193

5.8 Adaptive Grids 200
5.9 Some Modern Developments in Grid Generation 208
5.10 Some Modern Developments in Finite-Volume Mesh Generation: Unstructured Meshes and a Return to Cartesian Meshes 210
5.11 Summary 212
Problems 215

**6 Some Simple CFD Techniques: A Beginning** 216
6.1 Introduction 216
6.2 The Lax-Wendroff Technique 217
6.3 MacCormack's Technique 222

*GUIDEPOST* 223

6.4 Some Comments: Viscous Flows, Conservation Form, and Space Marching 225
6.4.1 Viscous Flows 225
6.4.2 Conservation Form 225
6.4.3 Space Marching 226
6.5 The Relaxation Technique and Its Use with Low-Speed Inviscid Flow 229
6.6 Aspects of Numerical Dissipation and Dispersion; Artificial Viscosity 232
6.7 The Alternating-Direction-Implicit (ADI) Technique 243
6.8 The Pressure Correction Technique: Application to Incompressible Viscous Flow 247
6.8.1 Some Comments on the Incompressible Navier–Stokes Equations 248

6.8.2 Some Comments on Central Differencing of the Incompressible Navier–Stokes Equations; The Need for a Staggered Grid 250
6.8.3 The Philosophy of the Pressure Correction Method 253
6.8.4 The Pressure Correction Formula 254
6.8.5 The Numerical Procedure: The SIMPLE Algorithm 261
6.8.6 Boundary Conditions for the Pressure Correction Method 262

*GUIDEPOST* 264

6.9 Some Computer Graphic Techniques Used in CFD 264
6.9.1 *xy* Plots 264
6.9.2 Contour Plots 265
6.9.3 Vector and Streamline Plots 270
6.9.4 Scatter Plots 273
6.9.5 Mesh Plots 273
6.9.6 Composite Plots 274
6.9.7 Summary on Computer Graphics 274
6.10 Summary 277
Problems 278

Part III Some Applications

**7 Numerical Solutions of Quasi-One-Dimensional Nozzle Flows** 283
7.1 Introduction: The Format for Chapters in Part III 283
7.2 Introduction to the Physical Problem: Subsonic-Supersonic Insentropic Flow 285
7.3 CFD Solution of Subsonic-Supersonic Isentropic Nozzle Flow: MacCormack's Technique 288
7.3.1 The Setup 288
7.3.2 Intermediate Results: The First Few Steps 308
7.3.3 Final Numerical Results: The Steady-State Solution 313
7.4 CFD Solution of Purely Subsonic Isentropic Nozzle Flow 325
7.4.1 The Setup: Boundary and Initial Conditions 327
7.4.2 Final Numerical Results: MacCormack's Technique 330
7.4.3 The Anatomy of a Failed Solution 325
7.5 The Subsonic-Supersonic Isentropic Nozzle Solution Revisited: The Use of the Governing Equations in Conservation Form 336
7.5.1 The Basic Equations in Conservation Form 337
7.5.2 The Setup 340
7.5.3 Intermediate Calculations: The First Time Step 345
7.5.4 Final Numerical Results: The Steady State Solution 351

7.6 A Case with Shock Capturing 356
7.6.1 The Setup 358
7.6.2 The Intermediate Time-Marching Procedure: The Need for Artificial Viscosity 363
7.6.3 Numerical Results 364
7.7 Summary 372

**8 Numerical Solution of a Two-Dimensional Supersonic Flow: Prandtl-Meyer Expansion Wave** 374
8.1 Introduction 374
8.2 Introduction to the Physical Problem: Prandtl-Meyer Expansion Wave—Exact Analytical Solution 376
8.3 The Numerical Solution of a Prandtl-Meyer Expansion Wave Flow Field 377
8.3.1 The Governing Equations 377
8.3.2 The Setup 386
8.3.3 Intermediate Results 397
8.3.4 Final Results 407
8.4 Summary 414

**9 Incompressible Couette Flow: Numerical Solutions by Means of an Implicit Method and the Pressure Correction Method** 416
9.1 Introduction 416
9.2 The Physical Problem and Its Exact Analytical Solution 417
9.3 The Numerical Approach: Implicit Crank-Nicholson Technique 420
9.3.1 The Numerical Formulation 421
9.3.2 The Setup 425
9.3.3 Intermediate Results 426
9.3.4 Final Results 430
9.4 Another Numerical Approach: The Pressure Correction Method 435
9.4.1 The Setup 436
9.4.2 Results 442
9.5 Summary 445
Problem 446

**10 Supersonic Flow over a Flat Plate: Numerical Solution by Solving the Complete Navier–Stokes Equations** 447
10.1 Introduction 447
10.2 The Physical Problem 449
10.3 The Numerical Approach: Explicit Finite-Difference Solution of the Two-Dimensional Complete Navier–Stokes Equations 450
10.3.1 The Governing Flow Equations 450
10.3.2 The Setup 452

10.3.3 The Finite-Difference Equations 453
10.3.4 Calculation of Step Sizes in Space and Time 455
10.3.5 Initial and Boundary Conditions 457
10.4 Organization of Your Navier–Stokes Code 459
10.4.1 Overview 459
10.4.2 The Main Program 461
10.4.3 The MacCormack Subroutine 463
10.4.4 Final Remarks 466
10.5 Final Numerical Results: The Steady State-Solution 466
10.6 Summary 474

Part IV Other Topics

11 Some Advanced Topics in Modern CFD: A Discussion 479
11.1 Introduction 479
11.2 The Conservation Form of the Governing Flow Equations Revisited: The Jacobians of the System 480
11.2.1 Specialization to One-Dimensional Flow 482
11.2.2 Interim Summary 489
11.3 Additional Considerations for Implicit Methods 489
11.3.1 Linearization of the Equations: The Beam and Warming Method 490
11.3.2 The Multidimensional Problem: Approximate Factorization 492
11.3.3 Block Tridiagonal Matrices 496
11.3.4 Interim Summary 497
11.4 Upwind Schemes 497
11.4.1 Flux-Vector Splitting 500
11.4.2 The Godunov Approach 502
11.4.3 General Comment 507
11.5 Second-Order Upwind Schemes 507
11.6 High-Resolution Schemes: TVD and Flux Limiters 509
11.7 Some Results 510
11.8 Multigrid Method 513
11.9 Summary 514
Problems 514

12 The Future of CFD 515
12.1 The Importance of CFD Revisited 515
12.2 Computer Graphics in CFD 516
12.3 The Future of CFD: Enhancing the Design Process 517
12.4 The Future of CFD: Enhancing Understanding 526
12.5 Conclusion 533

**Appendix A Thomas' Algorithm for the Solution of a Tridiagonal System of Equations** 534

**References** 539

Index 543