

Contents

1 Polymers	5
1.1 Introduction	5
1.2 The Domb-Joyce model	6
1.3 The Edwards model	10
1.4 Law of large numbers for the Domb-Joyce model	12
1.5 Law of large numbers for the Edwards model	15
1.6 Central limit theorem for the Domb-Joyce model	16
1.7 Organization of this work	16
1.8 Related models: an outlook on the future	18
1.8.1 Repulsion and attraction	18
1.8.2 Elasticity	19
1.8.3 Inhomogeneity	20
1.8.4 A polymer near an interface	20
1.8.5 Branching polymers	21
1.8.6 Conclusion: the scale of a random polymer	21
2 Scaling for a random polymer	23
2.1 Introduction and main results	23
2.1.1 Law of large numbers in Greven and den Hollander (1993): Theorem 2.1	23
2.1.2 Numerical estimates of $r^*(\beta)$ and $\theta^*(\beta)$	25
2.1.3 Main results for the weak interaction limit: Theorems 2.2–2.4	25
2.1.4 Figures illustrating Theorem 2.3	28
2.1.5 Relation to the Edwards model: Theorems 2.5–2.7	28
2.1.6 Outline of the proof	31
2.2 Preparations	32
2.2.1 A variational representation: Lemma 2.1	32
2.2.2 A key proposition: Proposition 2.1	33
2.2.3 Epi-convergence: Propositions 2.2–2.3	33
2.2.4 Properties of P : Lemmas 2.2–2.4	35
2.3 $(F_\beta^a)_{\beta>0}$ is epi-convergent to F^a : Lemmas 2.5–2.8	36
2.4 An approximate maximizer of F_β^a : Proposition 2.4	41
2.4.1 Proof of Proposition 2.4: Lemmas 2.9–2.12	41

2.5	Proof of Lemma 2.12	48
2.6	Analysis of the limit variational problem	48
2.6.1	Existence of a maximizer of F^a in \bar{K}_C^a : Lemma 2.13	49
2.6.2	Characterization of the minimizer(s) of \hat{F}^a : Lemma 2.14	51
2.6.3	Analysis of the Sturm-Liouville problem: Lemmas 2.15–2.17	52
2.6.4	Dependence on a : Lemma 2.18	54
2.7	Proof of Theorems 2.2–2.4	56
3	Central limit theorem for the Edwards model	61
3.1	The Edwards model in terms of Brownian local times	61
3.1.1	Main theorem: Theorem 3.1	62
3.1.2	Scaling in β	63
3.1.3	Outline of the proof	63
3.2	Reformulation of the problem	64
3.2.1	The main proposition: Proposition 3.1	64
3.2.2	Ray-Knight theorems for Brownian local times	64
3.2.3	The distribution of $(\{L(T, x)\}_{x \in \mathbb{R}}, B_T)$: Lemma 3.1	65
3.2.4	Application to the Edwards model: Lemma 3.2	69
3.3	Structure of the proof of Proposition 3.1	70
3.3.1	A transformed Markov process: Lemma 3.3	70
3.3.2	Properties of the transformed Markov process	71
3.3.3	Final reformulation: Lemma 3.4	73
3.3.4	Key steps in the proof of Proposition 3.1: Propositions 3.2–3.4	74
3.3.5	Proof of Proposition 3.1	76
3.4	CLT for the middle piece	77
3.4.1	Proof of Proposition 3.2	77
3.4.2	Proof of Proposition 3.3	78
3.5	Integrability for the boundary pieces	80
3.5.1	Preparations: Lemma 3.5	80
3.5.2	Proof of Proposition 3.4	81
4	CLT for a weakly interacting random polymer	85
4.1	Introduction and main results	85
4.1.1	Case $\beta_n \equiv \beta \downarrow 0$: Theorem 4.1	86
4.1.2	Case $\beta_n \rightarrow 0$ and $n^{\frac{3}{2}}\beta_n \rightarrow \infty$: Theorem 4.2	86
4.1.3	Discussion	87
4.1.4	Outline of the proof	88
4.2	Reformulation of the problem	88
4.2.1	The main proposition: Proposition 4.1	88
4.2.2	Knight's description of the local times	89
4.2.3	The distribution of $(\{\ell_n(x)\}_{x \in \mathbb{Z}}, S_n)$: Lemma 4.1	91
4.3	Structure of the proof of Proposition 4.1	93
4.3.1	A transformed Markov chain	93

4.3.2	A time-changed Markov chain	94
4.3.3	Unscaled representation: Lemma 4.2	95
4.3.4	Scaled representation: Lemma 4.3	98
4.3.5	A key proposition: Proposition 4.2	99
4.3.6	Proof of Proposition 4.1	99
4.4	Preparatory tools for the proof of Proposition 4.2	101
4.4.1	Spectral properties	101
4.4.2	Eigenvector scaling limits: Proposition 4.3	103
4.4.3	The function γ	104
4.4.4	Convergence of the function $w_{r,\beta}$: Lemmas 4.4–4.6	105
4.5	Proof of Proposition 4.2	106
4.5.1	Splitting the integrals: Lemmas 4.7–4.9	107
4.5.2	Proof of Lemma 4.7: cutting away large t_1, t_2	108
4.5.3	Proof of Lemma 4.8: cutting away small t_1, t_2	111
4.6	Proof of Lemma 4.9: intermediate t_1, t_2	114
4.6.1	Convergence along subsequences: Lemma 4.10	114
4.6.2	Identification of the limit: Lemma 4.11	117
4.7	Proof of Lemmas 4.4–4.6	121
4.7.1	Proof of Lemma 4.4: properties of \bar{w}_{r_n, β_n}	121
4.7.2	Preparations for the proof of Lemmas 4.5–4.6	123
4.7.3	Proof of Lemmas 4.5 and 4.6	126
4.8	Proof of Theorem 4.1	128
4.9	Proof of Proposition 4.3	130
4.9.1	Proof of Proposition 4.3(i): variational representations	131
4.9.2	Proof of Proposition 4.3(i): convergence of $\bar{\tau}_\beta^{(l)}$ and $\lambda^{(l)}(\beta)$	132
4.9.3	Proof of Proposition 4.3(ii): uniform convergence of $\bar{\tau}_\beta$	135
4.9.4	Proof of Proposition 4.3(iii)	137
5	The constants in the central limit theorems	139
5.1	Main theorem: Theorem 5.1	139
5.2	Preparations: Lemmas 5.1–5.4	140
5.2.1	Sturm-Liouville theory: Lemmas 5.1–5.3	140
5.2.2	Power series approximation: Lemma 5.4	143
5.3	Proof of Theorem 5.1(i)	144
5.4	Proof of Theorem 5.1(ii)	145
5.4.1	The lower bound for b^*	145
5.4.2	The upper bound for b^*	147
5.5	Proof of Theorem 5.1(iii)	148
5.5.1	The upper bound for c^* : Lemmas 5.5–5.6	148
5.5.2	Proof of Lemma 5.6: Spectral analysis of \mathcal{K}^*	149
5.5.3	The lower bound for c^*	150
5.6	Proof of Theorem 5.1(iv)	151

Bibliography	153
Author index	157
Summary	159
Acknowledgements	161
Glossary	163