Contents

Introduction 1

Chapter 1. The ℓ-adic Formalism 7
1. Quotients of t-Categories 7
2. ℓ-Adic Derived Categories 9
2.1. Functoriality 11

Chapter 2. Stratifications 15
1. d-Structures 15
2. cd-Structures 17

Chapter 3. Topoi 21
1. Fibered Topoi 21
2. Constructible Sheaves 24
3. Constructible ℓ-Adic Sheaves 26
4. Topoi with c-Structures and ℓ-Adic Derived Categories 29
5. The d-Structures Defined by c-Topoi 34
6. Topoi with e-Structures 37

Chapter 4. Algebraic Stacks 41
1. Preliminaries on Algebraic Stacks 41
1.1. Gerbe-Like morphisms 41
1.2. Devissage for Algebraic Stacks 43
1.3. Universal Homeomorphisms 45
2. The Étale Topos of an Algebraic Stack 46
2.1. The Flat Topos 46
2.2. The Étale Topos 47
3. The Smooth Topos 48
4. The Simplicial Approach 51
4.1. The d-structures defined by the simplicial c-topos 52
5. The Smooth Approach 53
6. The Étale-Smooth cd-Structure 57
7. The ℓ-Adic d-Structure 62
7.1. Closed L-Stratifications 62
7.2. Some more auxiliary results 64
7.3. The Main Theorem 65
8. Purity and Extraordinary Pullbacks 67

Chapter 5. Convergent Complexes 73
1. $\overline{Q}_\ell$-Complexes 73
1.1. A Theorem of Borel 75
2. Frobenius 75
3. Mixed and Convergent Complexes 78
3.1. Convergence 79
4. The Trace Formula 82
4.1. More about Frobenius 82
4.2. Trace of Frobenius 84
4.3. An Example 89

Bibliography 93