

Contents

Chapter 1. Introduction	1
Chapter 2. Heuristic discussion of the mechanism	7
2.1. Integrable systems, resonances, secondary tori	7
2.2. Heuristic description of the mechanism	9
Chapter 3. A simple model	15
Chapter 4. Statement of rigorous results	19
Chapter 5. Notation and definitions, resonances	25
Chapter 6. Geometric features of the unperturbed problem	27
Chapter 7. Persistence of the normally hyperbolic invariant manifold and its stable and unstable manifolds	31
7.1. Explicit calculations of the perturbed invariant manifold	33
Chapter 8. The dynamics in $\tilde{\Lambda}_\varepsilon$	37
8.1. A system of coordinates for $\tilde{\Lambda}_\varepsilon$	39
8.2. Calculation of the reduced Hamiltonian	41
8.3. Isolating the resonances (resonant averaging)	43
8.3.1. The infinitesimal equations for averaging	44
8.3.2. The main averaging result, Theorem 8.9	46
8.3.3. Proof of Theorem 8.9	47
8.4. The non-resonant region (KAM theorem)	50
8.4.1. Some results on Diophantine approximation	52
8.4.2. The KAM Theorem for twist maps	55
8.5. Analyzing the resonances	58
8.5.1. Resonances of order 3 and higher	58
8.5.2. Preliminary analysis of resonances of order one or two	59
8.5.3. Primary and secondary tori near the first and second order resonances	62
8.5.4. Proof of Theorem 8.30 and Corollary 8.31	68
8.5.5. Existence of stable and unstable manifolds of periodic orbits	82
Chapter 9. The scattering map	87
9.1. Some generalities about the scattering map	87
9.2. The scattering map in our model: definition and computation	89

Chapter 10. Existence of transition chains	97
10.1. Transition chains	99
10.2. The scattering map and the transversality of heteroclinic intersections	99
10.2.1. The non-resonant region and resonances of order 3 and higher	103
10.2.2. Resonances of first order	104
10.2.3. Resonances of order 2	110
10.3. Existence of transition chains to objects of different topological types	117
Chapter 11. Orbits shadowing the transition chains and proof of theorem 4.1	121
Chapter 12. Conclusions and remarks	123
12.1. The role of secondary tori and the speed of diffusion	123
12.2. Comparison with [DLS00]	123
12.3. Heuristics on the genericity properties of the hypothesis and the phenomena	124
12.4. The hypothesis of polynomial perturbations	125
12.5. Involving other objects	126
12.6. Variational methods	127
12.7. Diffusion times	127
Chapter 13. An example	129
Acknowledgments	135
Bibliography	137