	Preface ix To the Reader xiii Acknowledgments xv	
1	Following the Clues 1	
	Sample Problems 2 Which Chart or Diagram to Choose Presenting a Solution 11 Some Steps in Problem Solving 11 Tree Diagrams 16 The Multiplication Principle 19 Simplification 22 The Chapter in Retrospect 25	10
2	Exercises 26	
_	Solve It With Logic 37  Sample Problems 38  Statements 39  Variables and Connectives 41	
	Negation 41 "And"—Conjunction 42 "Or"—Disjunction 43 Conditional and Biconditional	
	Statements 44	
	Drawing Conclusions 47 Compound Statements 49	
	1	

Logical Implication and Equivalence 53 Arguments and Validity 55 The Chapter in Retrospect 60 Exercises 61
From Words to Equations: Algebraic Recreations 70
Sample Problems 71 Introducing Variables 72 The Chapter in Retrospect 84 Exercises 84
Solve It With Integers: Some Topics From Number Theory 100
Sample Problems 101 Diophantine Equations 103 Divisibility 104 Prime Numbers 106
☆The Infinitude of Primes 106
☆ The Sieve of Eratosthenes 107
More About Primes 108
Linear Diophantine Equations 115
Division With Remainders 119

Casting Out Nines 125 Solving Linear Congruences

127

	Solving Linear Diophantine Equations The Chapter in Retrospect 137 Exercises 138	131
5	More About Numbers: Bases and Cryptarithmetic	145
	Sample Problems 146 Positional Notation 147 Changing Bases 148 Addition and Multiplication in Other Bases 153	
	Cryptarithmetic 156 The Chapter in Retrospect 162 Exercises 163	
6	Solve It With Networks: An Introduction to Graph Theory 173	
	Sample Problems 174 Graphs 175 Eulerian Paths and Circuits 178 Odd and Even Vertices 180  ☆ More Than Two Odd Vertices 187  ☆ Directed Graphs 190 Hamiltonian Circuits 192 The Knight's Tour 194	
	Other Applications 196  ☆ Coloring Graphs and Maps 198  The Chapter in Retrospect 201  Exercises 202	
7	Games of Strategy for Two Players 213	
	Sample Problems 214 Chance-Free Decisionmaking 215 Games of Perfect Information 216 Finiteness 216 The Existence of Winning Strategies Position—State of the Game 224	217
	The State Diagram of a Game 228	

How Do We Find a Winning Strategy?	23C
Finding a Winning Strategy by Working Backward 230	
Finding Winning Strategies by Simplifying Game 232	ga
Finding Winning Strategies With a Fronto Assault 233	lc
How Many Possibilities Need Be Considered? 234	
Symmetry as a Limiting Factor 234	
Déjà Vu-We've Seen It Before 241	
The Game of Nim 246	
Pairing Strategies 250	
Variations of a Game 251	
The Chapter in Retrospect 252	
Exercises 253	
200	
Solitaire Games and	
Puzzles 273	
Sample Problems 274	
The Tower of Brahma 276	
Dissection Problems 279	
Polyominoes 281	
Soma 284	
Peg Solitaire 285	
The Fifteen Puzzle 288	
Even and Odd Permutations 291	
☆ Coloring and the 15 Puzzle—A Second	
Approach 298	
Colored Cubes 299	
☆ Colored Cubes—A Second Approach 303	
The Chapter in Retrospect 304	
Exercises 305	
2.45.5.555	
Potpourri 313	
Decimation 313	
Coin Weighing 315	
Shunting 316	
Syllogisms 317	
Grab Bag 318	
The Book in Retrospect 320	

## Appendix A—Some Basic Algebraic Techniques 32<sup>o</sup> Appendix B—Mathematical Induction 332 Bibliography 336

## Hints and Solutions 34 Answers to Selected Problems 381 Index 397