

# CONTENTS

PROBLEM	PAGE
1. The Chess Tournament. . . . .	1
2. The Ordered Partitions of $n$ . . . . .	3
3. Regions in a Circle. . . . .	4
4. The Ferry Boats. . . . .	7
5. The Bulging Semicircle. . . . .	8
6. The Chauffeur Problem. . . . .	10
7. The Screens in the Corner. . . . .	13
8. Coloring the Plane. . . . .	15
9. An Obvious Maximum. . . . .	16
10. $\cos 17x = f(\cos x)$ . . . . .	18
11. A Square in a Lattice. . . . .	19
12. An Opaque Square. . . . .	22
13. $\times$ 's and O's. . . . .	26
14. A Surprising Property of Right-Angled Triangles. . . . .	27
15. The Digits of $4444^{4444}$ . . . . .	30
16. $\sigma(n) + \varphi(n) = n \cdot d(n)$ . . . . .	32
17. On $k$ -Clouds. . . . .	35
18. A Minimal Sum. . . . .	37
19. The Last Three Digits of $7^{9999}$ . . . . .	40
20. Rolling a Die. . . . .	42
21. Piercing a Cube. . . . .	43
22. Double Sequences. . . . .	45
23. Point-Splitting Circles. . . . .	48

24. On the Lengths of the Sides of a Triangle. . . . .	52
25. No Calculus Please. . . . .	53
26. $a^b$ and $b^a$ . . . . .	59
27. A Mathematical Joke. . . . .	61
28. Maps on a Sphere. . . . .	63
29. Convex Regions in the Plane. . . . .	66
30. Simultaneous Diophantine Equations. . . . .	71
31. A Reflected Tangent. . . . .	72
32. The Elegantly Destroyed Checkerboard. . . . .	74
33. The Snowballs. . . . .	79
34. Writing the Numbers from One to a Billion. . . . .	81
35. Abutting, Nonoverlapping Unit Squares. . . . .	82
36. A Diophantine Equation. . . . .	90
37. The Fibonacci Sequence. . . . .	92
38. An Erdős Inequality. . . . .	96
39. Sharing Lattice Points. . . . .	99
40. Perfect Numbers. . . . .	102
41. The Sides of a Quadrilateral. . . . .	105
42. Primes in Arithmetic Progression. . . . .	107
43. On Cevians. . . . .	109
44. The Cows and the Sheep. . . . .	112
45. A Sequence of Squares. . . . .	114
46. The Inscribed Decagon. . . . .	115
47. Red and Blue Dots. . . . .	118
48. Swale's Method. . . . .	121
49. On $\pi(n)$ . . . . .	123
50. A Constant Chord. . . . .	126
51. The Number of Inner Diagonals. . . . .	128

52. Loading the Dice. . . . .	130
53. A Curious Sequence. . . . .	132
54. Long Strings of Consecutive Natural Numbers. . . . .	136
55. A Minimal Inscribed Quadrilateral. . . . .	138
56. Triangular Numbers. . . . .	142
57. On a Regular $n$ -gon. . . . .	150
58. The Fermat Numbers. . . . .	152
59. An Inequality of Reciprocals. . . . .	155
60. A Perfect 4th Power. . . . .	156
61. Packing Squares. . . . .	157
62. Red and Green Balls. . . . .	163
63. Composite Terms in Arithmetic Progression. . . . .	165
64. Abutting Equilateral Triangles. . . . .	167
65. The Tests. . . . .	170
66. An Application of Ptolemy's Theorem. . . . .	172
67. Another Diophantine Equation. . . . .	176
68. An Unusual Property of Complex Numbers. . . . .	178
69. A Chain of Circles. . . . .	180
70. Repeated Digits at the End of a Square. . . . .	184
71. An Angle Bisector. . . . .	186
72. A System of Inequalities. . . . .	188
73. An Unexpected Property of a Regular 26-gon . . . . .	189
74. More on Perfect Squares. . . . .	192
75. An Unusual Polynomial. . . . .	196
76. Cyclic Centroids. . . . .	198
77. An Easy Remainder. . . . .	202
78. A Curious Property of 3. . . . .	203
79. A Square Within a Square. . . . .	204

80. Always a Square. . . . .	206
81. Grouping the Natural Numbers. . . . .	207
82. Triangles with Sides in Arithmetic Progression. . . . .	209
83. Fractions by Permutation. . . . .	211
84. On Binomial Coefficients. . . . .	212
85. The Fermat Number $F_{73}$ . . . . .	214
86. A Cyclic Quadrilateral. . . . .	218
87. Special Triples of Natural Numbers. . . . .	220
88. The Sums of the Primes. . . . .	221
89. Another Curious Sequence. . . . .	223
90. The Ellipse and the Lattice. . . . .	228
91. Archimedes Triangles. . . . .	236
Exercises. . . . .	243
List of the Problems by Topic. . . . .	247