

## CONTENTS

Preface to the Tenth Edition	v
Preface to the Eleventh Edition	vii
Preface to the Twelfth Edition	ix
I ARITHMETICAL RECREATIONS	3
To find a number selected by someone	5
Prediction of the result of certain operations	8
Problems involving two numbers	11
Problems depending on the scale of notation	12
Other problems with numbers in the denary scale	14
Four fours problem	16
Problems with a series of numbered things	17
Arithmetical restorations	20
Calendar problems	26
Medieval problems in arithmetic	27
The Josephus problem. Decimation	32
Nim and similar games	36
Moore's game	38
Kayles	39
Wythoff's game	39
Addendum on solutions	40
II ARITHMETICAL RECREATIONS ( <i>continued</i> )	41
Arithmetical fallacies	41
Paradoxical problems	44
Probability problems	45
Permutation problems	48
Bachet's weights problem	50
The decimal expression for $1/n$	53
Decimals and continued fractions	54

Rational right-angled triangles	57
Triangular and pyramidal numbers	59
Divisibility	60
The prime number theorem	62
Mersenne numbers	64
Perfect numbers	66
Fermat numbers	67
Fermat's Last Theorem	69
Galois fields	73
III GEOMETRICAL RECREATIONS	76
Geometrical fallacies	76
Geometrical paradoxes	84
Continued fractions and lattice points	86
Geometrical dissections	87
Cyclotomy	94
Compass problems	96
The five-disc problem	97
Lebesgue's minimal problem	99
Keakeya's minimal problem	99
Addendum on a solution	102
IV GEOMETRICAL RECREATIONS ( <i>continued</i> )	103
Statical games of position	103
Three-in-a-row. Extension to $p$ -in-a-row	103
Tessellation	105
Anallagmatic pavements	107
Polyominoes	109
Colour-cube problem	113
Squaring the square	115
Dynamical games of position	116
Shunting problems	116
Ferry-boat problems	118
Geodesic problems	120
Problems with counters or pawns	121
Paradromic rings	127
Addendum on solutions	129

V	POLYHEDRA	130
	Symmetry and symmetries	130
	The five Platonic solids	131
	Kepler's mysticism	133
	Pappus, on the distribution of vertices	134
	Compounds	135
	The Archimedean solids	136
	Mrs. Stott's construction	139
	Equilateral zonohedra	141
	The Kepler-Poinsot polyhedra	144
	The 59 icosahedra	146
	Solid tessellations	147
	Ball-piling or close-packing	149
	The sand by the sea-shore	151
	Regular sponges	152
	Rotating rings of tetrahedra	154
	The kaleidoscope	155
VI	CHESS-BOARD RECREATIONS	162
	Relative value of pieces	163
	The eight queens problem	166
	Maximum pieces problem	172
	Minimum pieces problem	172
	Re-entrant paths on a chess-board	175
	Knight's re-entrant path	175
	King's re-entrant path	186
	Rook's re-entrant path	187
	Bishop's re-entrant path	187
	Routes on a chess-board	187
	Guarini's problem	189
	Latin squares	189
	Eulerian squares	190
	Euler's officers problem	192
	Eulerian cubes	192
VII	MAGIC SQUARES	193
	Magic squares of an odd order	195

xiv CONTENTS

Magic squares of a singly-even order	196
Magic squares of a doubly-even order	199
Bordered squares	200
Number of squares of a given order	201
Symmetrical and pandiagonal squares	202
Generalization of De la Loubère's rule	204
Arnoux's method	206
Margossian's method	207
Magic squares of non-consecutive numbers	210
Magic squares of primes	211
Doubly-magic and trebly-magic squares	212
Other magic problems	213
Magic domino squares	213
Cubic and octahedral dice	214
Interlocked hexagons	215
Magic cubes	216
VIII MAP-COLOURING PROBLEMS	222
The four-colour conjecture	222
The Petersen graph	225
Reduction to a standard map	227
Minimum number of districts for possible failure	230
Equivalent problem in the theory of numbers	231
Unbounded surfaces	232
Dual maps	234
Maps on various surfaces	234
Pits, peaks, and passes	238
Colouring the icosahedron	238
IX UNICURSAL PROBLEMS	243
Euler's problem	243
Number of ways of describing a unicursal figure	250
Mazes	254
Trees	260
The Hamiltonian game	262
Dragon designs	266

## X COMBINATORIAL DESIGNS 271

- A projective plane 271
- Incidence matrices 272
- An Hadamard matrix 273
- An error-correcting code 274
- A block design 276
- Steiner triple systems 278
- Finite geometries 281
- Kirkman's school-girl problem 287
- Latin squares 290
- The cube and the simplex 295
- Hadamard matrices 296
- Picture transmission 297
- Equiangular lines in 3-space 299
- Lines in higher-dimensional space 303
- C-matrices 308
- Projective planes 310

## XI MISCELLANEOUS PROBLEMS 312

- The fifteen puzzle 312
- The Tower of Hanoi 316
- Chinese rings 318
- Problems connected with a pack of cards 322
- Shuffling a pack 323
- Arrangements by rows and columns 325
- Bachet's problem with pairs of cards 326
- Gergonne's pile problem 328
- The window reader 333
- The mouse trap. Treize 336

## XII THREE CLASSICAL GEOMETRICAL PROBLEMS 338

- The duplication of the cube 339
  - Solutions by Hippocrates, Archytas, Plato, Menaechmus,  
Apollonius, and Diocles 341
  - Solutions by Vieta, Descartes, Gregory of St. Vincent, and  
Newton 343

The trisection of an angle	344
Solutions by Pappus, Descartes, Newton, Clairaut, and Chasles	334
The quadrature of the circle	347
Origin of symbol $\pi$	349
Geometrical methods of approximation to the numerical value of $\pi$	349
Results of Egyptians, Babylonians, Jews	350
Results of Archimedes and other Greek writers	351
Results of European writers, 1200–1630	352
Theorems of Wallis and Brouncker	355
Results of European writers, 1699–1873	356
Approximations by the theory of probability	359
XIII CALCULATING PRODIGIES	360
John Wallis, 1616–1703	361
Buxton, circ. 1707–1772	361
Fuller, 1710–1790; Ampère	364
Gauss, Whately	365
Colburn, 1804–1840	365
Bidder, 1806–1878	367
Mondeux, Mangiamele	372
Dase, 1824–1861	372
Safford, 1836–1901	374
Zamebone, Diamandi, Rückle	375
Inaudi, 1867-	375
Types of memory of numbers	377
Bidder's analysis of methods used	378
Multiplication	379
Digital method for division and factors	381
Square roots. Higher roots	382
Compound interest	384
Logarithms	385
Alexander Craig Aitken	386
XIV CRYPTOGRAPHY AND CRYPTANALYSIS	388
Cryptographic systems	389

Transposition systems	391
Columnar transposition	392
Digraphs and trigraphs	394
Comparison of several messages	397
The grille	401
Substitution systems	402
Tables of frequency	404
Polyalphabetic systems	406
The Vigenère square	407
The Playfair cipher	410
Code	412
Determination of cryptographic system	414
A few final remarks	416
Addendum: References for further study	418
INDEX	419