

Contents

Preface	vii
Chapter 3. The Legacy of Trigonometric Functions	1
Introduction	3
Lecture 1. Trigonometric Functions and Infinite Series	5
1.1. The Birth of Sine (\sin), a Trigonometric Ratio	5
1.2. The Trigonometric Function: $\sin x$	8
1.3. $\sin^{-1} x$ and $\tan^{-1} x$	11
1.4. Integral Representations and Power Series Representations of Functions	13
1.5. Reversing Infinite Power Series	21
1.6. History	25
Lecture 2. Elliptic Functions	29
2.1. Ellipses	29
2.2. Lemniscates	35
2.3. Multiple-Angle Formulas	40
2.4. The Addition Formula for the Lemniscate Sine	46
2.5. Arithmetic and Geometric Means	48
Chapter 4. Intersection of Geometry and Algebra	57
Introduction	59
Lecture 1. The Poncelet Closure Theorem	63
1.1. Poncelet Theorem—the Basics	63
1.2. The Poncelet Theorem	66
1.3. Introduction of the Algebraic Point of View	73
1.4. The Proof of the Poncelet Closure Theorem	76
Lecture 2. The Poncelet Theorem for Circles	81
2.1. The Case of Circles	81
2.2. The Jacobian Invariant	82
2.3. The Analytic Proof of the Poncelet Theorem	88
Lecture 3. The Poncelet Theorem in the World of Complex Numbers	91

3.1.	Introducing Complex Numbers	91
3.2.	A Point at Infinity	98
3.3.	Projective Planes	102
3.4.	Tangent Lines to Plane Curves of Degree 2	109
3.5.	Unit Circles and Lines in $\mathbb{P}^2(\mathbb{C})$	112
Appendix.	Proof of the Poncelet Theorem Using Plane Geometry	117
A.1.	Coaxal Circles	117
A.2.	The Plane Geometry Proof of the Poncelet Theorem	124
Conclusion		127