CONTENTS

1. NUMBERS AND NUMBER SYSTEMS 1

A. Sets and sub-sets 1
B. One-to-one correspondence 3
C. Cardinal numbers 4
D. Counting 7
E. Number systems 9

2. OPERATIONS WITH SETS AND NUMBERS 15

A. The union and intersection of sets 15
   1. The union and intersection of two sets 15
   2. The union and intersection of more than two sets 18
B. Operations with numbers 22
   1. Addition 22
      (a) The addition of two natural numbers 22
      (*b*) The addition of more than two natural numbers 24
   2. Multiplication 26
   3. Subtraction 28
   4. Division 29
C. Properties of sets and number systems 30
   1. The commutative property 30
   2. The associative property 32
   3. The distributive property 33
D. Identity elements 34

3. WRITTEN RECKONING 36

A. Addition 37
B. Multiplication 39
C. Subtraction 44
D. Division 45
E. The use of the index notation in written reckoning 48
   1. The index notation 48
   2. Using the index notation in multiplication and division 50

4. GEOMETRICAL FORM 51

A. The natural occurrence of mathematical shapes 51
B. Naming some of the common shapes 54
C. Breaking down and constructing some shapes 56

5. PLANE SHAPES 58

A. Familiar plane shapes 58
B. Ways in which plane shapes arise 59
   1. Plane shapes arising from motion 60
   2. Plans, elevations, and sections 62
   3. Plane shapes arising from sets of straight lines 66

6. A SET OF NEW NUMBERS 70
A. The need for new numbers 70
B. Introducing the new numbers 71
C. Operations with the integers 72
1. The addition operation 73
2. The subtraction operation 75
3. The multiplication operation 76
4. The division operation 78
5. Summary 79
D. Inequalities 80

7. FRACTIONAL PARTS 84
A. Common fractions 84
B. Decimal fractions 87

8. ANOTHER SET OF NEW NUMBERS 90
A. The need for more new numbers 90
B. Operations with the rational numbers 93
1. The multiplication operation 93
2. The division operation 96
3. The addition operation 96
4. The subtraction operation 99
C. The rationals and number lines 101
D. Inequalities 107
E. Another look at the number line 108
F. Using number lines in graphical work 110
G. Extending the use of the index notation 114

9. MEASURING AND THE METRIC SYSTEM 116
A. The origins of the metric system 116
B. Using the metric units 117
1. Length 117
2. Area 122
3. Volume 128
4. Weight 133
5. Capacity 134
C. Some other measures 135
1. Angular measure 135
2. Time 145
D. The complete metric notation 147

10. MOVING AND CHANGING SHAPES 149
A. Translations 149
1. Translating points 149
2. Translating sets of points 152
B. Rotations 159
1. Rotating points 159
2. Rotating sets of points 160

C. Reflections 164
1. Reflection in a straight line 164
2. Reflections in a pair of parallel lines 168
3. Reflections in a pair of intersecting lines 173
D. Summary 179

11. A PATTERN EMERGES 180
A. More about sets and operations 180
1. Interchanging the wheels of a car 180
2. Moving the wheels round 184
3. Rotating a star 186
B. An emerging pattern 189

ANSWERS 191

INDEX 211

PREFACE

This book has been written for students and teachers who wish to become more familiar with present-day developments in school mathematics. It attemps to present a unified approach to the subject.

The general aim of Part I is to introduce something of the new spirit in school mathematics without over-emphasizing mathematical rigour. In Part II the work is developed in such a way as to bring out the underlying structure of the subject.

Through the many practical activities and exercises the readers will be able to make progress by their own discoveries. Thus, while the content is primarily concerned with mathematics itself, the approach used indicates ways of developing the work in the classroom.

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