

Contents

| | | |
|---|--|---------|
| | <i>Preface</i> | page xi |
| | <i>List of notation</i> | xix |
| 1 | The algebra of polynomials | 1 |
| | 1.1 Complex polynomials | 1 |
| | 1.2 The number of zeros of a real analytic polynomial | 4 |
| | 1.3 Real analytic polynomials at infinity | 13 |
| 2 | The degree principle and the fundamental theorem of algebra | 22 |
| | 2.1 The fundamental theorem of algebra | 22 |
| | 2.2 Continuous functions in the plane | 26 |
| | 2.3 The degree principle | 31 |
| | 2.4 The degree principle and homotopy | 40 |
| | 2.5 The topological argument principle | 43 |
| | 2.6 The coincidence theorem | 47 |
| | 2.7 Locally 1–1 functions | 56 |
| | 2.8 The Borsuk–Ulam theorem | 79 |
| 3 | The Jacobian problem | 81 |
| | 3.1 The Jacobian conjecture | 81 |
| | 3.2 Pinchuk’s example | 90 |
| | 3.3 Polynomials with a constant Jacobian | 105 |
| | 3.4 A topological approach | 118 |
| | 3.5 The resultant and the Jacobian | 124 |
| 4 | Analytic and harmonic functions in the unit disc | 125 |
| | 4.1 Series representations | 125 |
| | 4.2 Positive and bounded operators | 138 |
| | 4.3 Positive trigonometric polynomials | 144 |
| | 4.4 Some inequalities for analytic and trigonometric polynomials | 151 |

| | | |
|-----|---|-----|
| 4.5 | Cesàro means | 156 |
| 4.6 | De la Vallée Poussin means | 160 |
| 4.7 | Integral representations | 167 |
| 4.8 | Generalised convolution operators | 168 |
| 5 | Circular regions and Grace's theorem | 172 |
| 5.1 | Convolutions and duality | 172 |
| 5.2 | Circular regions | 178 |
| 5.3 | The polar derivative | 184 |
| 5.4 | Locating critical points | 186 |
| 5.5 | Critical points of rational functions | 190 |
| 5.6 | The Borwein–Erdélyi inequality | 193 |
| 5.7 | Univalence properties of polynomials | 196 |
| 5.8 | Linear operators | 203 |
| 6 | The Ilieff–Sendov conjecture | 206 |
| 6.1 | Introduction | 206 |
| 6.2 | Proof of the conjecture for those zeros on the unit circle | 207 |
| 6.3 | The direct application of Grace's theorem | 208 |
| 6.4 | A global upper bound | 213 |
| 6.5 | Inequalities relating the nearest critical point to the nearest second zero | 216 |
| 6.6 | The extremal distance | 221 |
| 6.7 | Further remarks on the conjecture | 223 |
| 7 | Self-inversive polynomials | 228 |
| 7.1 | Introduction | 228 |
| 7.2 | Polynomials with interspersed zeros on the unit circle | 232 |
| 7.3 | Relations with the maximum modulus | 238 |
| 7.4 | Univalent polynomials | 241 |
| 7.5 | A second necessary and sufficient condition for angular separation of zeros | 249 |
| 7.6 | Suffridge's extremal polynomials | 251 |
| 8 | Duality and an extension of Grace's theorem to rational functions | 263 |
| 8.1 | Linear operators and rational functions | 263 |
| 8.2 | Interpretations of the convolution conditions | 270 |
| 8.3 | The duality theorem for $T(1, \beta)$ | 275 |
| 8.4 | The duality theorem for $T(m, \beta)$ | 282 |
| 8.5 | The duality principle | 286 |
| 8.6 | Duality and the class $T(\alpha, \beta)$ | 289 |
| 8.7 | Properties of the Kaplan classes | 293 |
| 8.8 | The class $S(\alpha, \beta)$ | 296 |

| | | |
|------|--|-----|
| 8.9 | The classes $T_0(\alpha, \beta)$ | 300 |
| 8.10 | The class $T(2, 2)$ | 302 |
| 9 | Real polynomials | 304 |
| 9.1 | Real polynomials | 304 |
| 9.2 | Descartes' rule of signs | 317 |
| 9.3 | Strongly real rational functions | 319 |
| 9.4 | Critical points of real rational functions | 323 |
| 9.5 | Rational functions with real critical points | 325 |
| 9.6 | Real entire and meromorphic functions | 326 |
| 10 | Level curves | 350 |
| 10.1 | Level regions for polynomials | 350 |
| 10.2 | Level regions of rational functions | 353 |
| 10.3 | Partial fraction decomposition | 355 |
| 10.4 | Smale's conjecture | 358 |
| 11 | Miscellaneous topics | 370 |
| 11.1 | The <i>abc</i> theorem | 370 |
| 11.2 | Cohn's reduction method | 372 |
| 11.3 | Blaschke products | 373 |
| 11.4 | Blaschke products and harmonic mappings | 377 |
| 11.5 | Blaschke products and convex curves | 382 |
| 11.6 | Blaschke products and convex polygons | 392 |
| 11.7 | The mapping problem for Jordan polygons | 402 |
| 11.8 | Sudbery's theorem on zeros of successive derivatives | 407 |
| 11.9 | Extensions of Sudbery's theorem | 413 |
| | <i>References</i> | 416 |
| | <i>Index</i> | 421 |