

Contents

	<i>Page</i>
<i>Preface to the second edition</i>	vii
<i>From the preface to the first edition</i>	ix
<i>Note to the reader</i>	xvii
<i>Some terminology, notations and conventions used throughout the book</i>	xviii
<i>List of special notation</i>	xxi
CHAPTER 0 PRELIMINARIES	1
0.1 Matrix rings	1
0.2 Rank conditions on free modules	6
0.3 Projective modules	10
0.4 Hermite rings	15
0.5 The monoid of projectives	19
0.6 The matrix of definition of a module	23
0.7 Eigenrings and centralizers	30
0.8 Groups and rings of fractions	34
0.9 Modules over Ore domains	46
0.10 Skew polynomial rings	50
0.11 Free associative algebras and tensor rings	59
Notes and comments	62
CHAPTER 1 FIRS, SEMIFIRS AND n-FIRS	64
1.1 Dependence relations	64
1.2 Firs and α -firs	71
1.3 Strong G_n -rings	74
1.4 Homological properties of firs and semifirs	77
1.5 Further properties of n -firs	80
1.6 Inert extensions	83
Notes and comments	85
CHAPTER 2 THE WEAK ALGORITHM	87
2.1 The division algorithm	87
2.2 The n -term weak algorithm	94

2.3	The associated graded ring	100
2.4	Weak algebra bases in filtered rings and free algebras	104
2.5	The Hilbert series of a filtered ring.	107
2.6	The construction of rings with a weak algorithm	111
2.7	Generators and relations for $\mathbf{GE}_2(R)$	115
2.8	The 2-term weak algorithm	121
2.9	The inverse weak algorithm	124
2.10	The transfinite weak algorithm	138
2.11	Computation of the dependence number	143
	Notes and comments	150
 CHAPTER 3 FACTORIZATION		153
3.1	The commutative case	153
3.2	Similarity in semifirs	158
3.3	Factorization in matrix rings over semifirs	164
3.4	Rigid factorizations	170
3.5	Factorization in semifirs: a closer look	177
3.6	The primary decomposition	184
	Notes and comments	191
 CHAPTER 4 RINGS WITH A DISTRIBUTIVE FACTOR LATTICE		193
4.1	Distributive modules	193
4.2	Distributive factor lattices	199
4.3	Conditions for a distributive factor lattice	204
4.4	Finite distributive lattices	211
4.5	The factor lattice	215
4.6	Eigenrings	219
	Notes and comments	226
 CHAPTER 5 MODULES OVER FIRS AND SEMIFIRS		227
5.1	Bound and unbound modules	227
5.2	Duality.	232
5.3	Positive and negative modules over semifirs	235
5.4	The ranks of matrices	245
5.5	Sylvester domains.	252
5.6	Stable inner rank and pseudo-Sylvester domains	261
5.7	The factorization of matrices over semifirs	265
5.8	A normal form for matrices over a free algebra	272

5.9	The specialization lemma	281
5.10	Ascending chain conditions	287
5.11	The intersection theorem for firs	294
	Notes and comments	296
 CHAPTER 6 CENTRALIZERS, SUBALGEBRAS AND AUTOMORPHISMS		 298
6.1	Commutative subrings and central elements in 2-firs	298
6.2	2-Firs with prescribed centre.	305
6.3	The centre of a fir	309
6.4	Bounded elements in 2-firs	312
6.5	Free monoids	321
6.6	Subalgebras and ideals of free algebras	329
6.7	Centralizers in power series rings and in free algebras	337
6.8	Automorphisms of polynomial rings	341
6.9	Automorphisms of free algebras	352
6.10	Invariants in free algebras	362
6.11	The Galois correspondence	371
	Notes and comments	378
 CHAPTER 7 SKEW FIELDS OF FRACTIONS		 381
7.1	The rational closure of a homomorphism	381
7.2	The category of R -fields and specializations	388
7.3	Matrix ideals	395
7.4	Prime matrix ideals	400
7.5	Fields of fractions	412
7.6	Numerators and denominators	421
7.7	The depth	432
7.8	Centralizers in universal fields of fractions of firs	437
7.9	Determinants and valuations	445
7.10	Localization of semifirs and firs	453
7.11	Universal localization: another route	462
	Notes and comments	486
 CHAPTER 8 PRINCIPAL IDEAL DOMAINS		 489
8.1	The diagonal reduction for matrices over a principal ideal domain.	489
8.2	Finitely generated modules over principal ideal domains	492
8.3	Skew polynomial rings	497

8.4	Theory of a single pseudo-linear transformation	503
8.5	Eigenvalues and algebraic matrices	506
8.6	Algebraic skew field extensions	515
8.7	Laurent series and the Malcev–Neumann construction	521
8.8	Iterated skew polynomial rings	532
	Notes and comments	538

APPENDIX

1.	Lattice theory	540
2.	Categories and homological algebra	544
	Bibliography and author index	557
	Subject index	581