

CONTENTS

Interdependence guide.....	vi
Preface	vii
<u>§ 0 PREREQUISITES.....</u>	1
 <u>PART I: NEAR-RINGS FOR BEGINNERS</u>	
<u>§ 1 THE ELEMENTARY THEORY OF NEAR-RINGS.....</u>	6
a) Fundamental definitions and properties.....	7
1) Near-rings.....	7
2) N-groups.....	13
3) Substructures.....	14
4) Homomorphisms and ideal-like subsets.....	15
5) Annihilators.....	20
6) Generated objects.....	23
b) Constructions.....	24
1) Products, direct sums and subdirect products.....	24
2) Near-rings of quotients.....	26
3) Free near-rings and N-groups.....	29
c) Embeddings.....	33
1) Embedding in $M(\Gamma)$	33
2) More beds.....	37
d) Some axiomatic considerations.....	38
1) Miscellaneous results.....	38
2) Related structures.....	41
<u>§ 2 IDEAL THEORY.....</u>	43
a) Sums.....	44
1) Sums and direct sums.....	44
2) Distributive sums.....	49
b) Chain conditions.....	50
c) Decomposition theorems.....	53
d) Prime ideals.....	61
1) Products of subsets.....	61
2) Prime ideals.....	62
3) Semiprime ideals.....	66
e) Nil and nilpotent.....	69

PART II: STRUCTURE THEORY

<u>§ 3 ELEMENTS OF THE STRUCTURE THEORY.....</u>	74
a) Types of N-groups.....	75
b) Change of the near-ring.....	81
c) Modularity.....	84
d) Quasiregularity.....	89
e) Idempotents.....	91
f) More on minimality.....	95
<u>§ 4 PRIMITIVE NEAR-RINGS.....</u>	102
a) General.....	103
1) Definitions and elementary results.....	103
2) The centralizer.....	106
3) Independence and density.....	110
b) 0-primitive near-rings.....	115
c) 1-primitive near-rings.....	120
d) 2-primitive near-rings.....	124
1) 2-primitive near-rings.....	124
2) 2-primitive near-rings with identity.....	126
3) 2-primitive zero-symmetric near-rings with identity and a minimal left ideal.....	130
4) 2-primitive near-rings with identity and minimum condition.....	131
5) An application to interpolation theory.....	133
<u>§ 5 RADICAL THEORY.....</u>	135
a) Jacobson-type radicals: common theory.....	136
1) Definitions and characterizations of the radicals.....	136
2) Radicals of related near-rings.....	139
3) Semisimplicity.....	145
b) Jacobson-type radicals: special theory.....	149
1) \mathfrak{J}_0 and $\mathfrak{J}_{1/2}$	149
2) \mathfrak{J}_1	152
3) \mathfrak{J}_2	152
c) The nil radical.....	160
d) The prime radical.....	161
e) Conclusive remarks.....	163

PART III: SPECIAL CLASSES OF NEAR-RINGS

<u>§ 6 DISTRIBUTIVELY GENERATED NEAR-RINGS.....</u>	170
a) Elementary.....	171
b) Some axiomatics.....	174
c) Constructions of distributively generated near-rings.....	176
d) Distributively generated near-rings with finiteness conditions.....	178
e) "Free" distributively generated near-rings.....	180
f) D-groups and (N,D)-groups.....	182
g) Structure theory.....	184

<u>§ 7 TRANSFORMATION NEAR-RINGS</u>	188
a) $M_H^0(\Gamma)$	189
b) $M(\Gamma)$ and $M_0(\Gamma)$	197
c) $E(\Gamma)$, $A(\Gamma)$ and $I(\Gamma)$	206
d) Polynomial near-rings.....	215
1) Polynomials and polynomial functions.....	215
2) $R[x]$	218
3) $\bar{R}[x]$	219
4) Ideal theory in $R[x]$	220
5) $F[x]$	223
6) $\Gamma[x]$ and $\bar{\Gamma}[x]$	230
7) Concluding remarks.....	232
 <u>§ 8 NEAR-FIELDS AND PLANAR NEAR-RINGS</u>	 236
a) Near-fields.....	237
1) Conditions to be a near-field.....	237
2) The additive group of a near-field.....	239
3) The center and the kernel of a near-field.....	241
4) Dickson near-fields.....	242
5) Near-fields and doubly transitive groups.....	246
6) Normal near-fields and incidence groups.....	248
7) Planar near-fields.....	253
b) Planar near-rings.....	256
1) The structure of planar near-rings.....	256
2) Planar near-rings and Balanced Incomplete Block Designs.....	264
 <u>§ 9 MORE CLASSES OF NEAR-RINGS</u>	 275
a) IFP-near-rings.....	276
1) IFP-near-rings.....	276
2) p-near-rings.....	286
3) Boolean near-rings.....	288
b) Near-rings without.....	289
1) Near-rings without nilpotent elements.....	289
2) Near-rings without zero divisors (integral near- rings).....	293
c) Affine near-rings.....	301
d) Near-rings on given groups.....	309
1) Multiplications on a group.....	309
2) Near-rings on simple and on cyclic groups.....	312
3) Near-rings with identities on given groups.....	314
4) Near-rings with other properties on given groups.....	317
e) Ordered near-rings.....	319
f) Miscellaneous topics.....	330

<u>APPENDIX</u>	337
Near-rings of low order.....	338
List of open problems.....	349
Bibliography.....	350
Supplementary works.....	387
List of symbols and abbreviations.....	388
Index.....	390