

TABLE OF CONTENTS

CHAPTER		PAGE
I. FUNDAMENTAL CONCEPTS		
1.	1. The notations	1
2.	2. Linear sets over \mathfrak{F}	1
3.	3. Algebras over \mathfrak{F}	2
4.	4. Products of linear subsets of an algebra	3
5.	5. Direct products	5
6.	6. The \mathfrak{A} -commutator of a subset	6
7.	7. Total matrix algebras	6
8.	8. Automorphisms of an algebra	7
9.	9. Linear transformations	9
10.	10. Regular quantities of an algebra	13
11.	11. Division algebras	13
12.	12. Scalar extension	15
13.	13. The minimum function of a division algebra	16
14.	14. The norm and trace functions	18
15.	15. A theorem of Wedderburn	19
II. IDEALS AND NILPOTENT ALGEBRAS		
1.	1. Idempotent quantities of an algebra	20
2.	2. Left ideals	21
3.	3. Ideals of \mathfrak{A}	22
4.	4. Nilpotent algebras	22
5.	5. The radical of an algebra	22
6.	6. The existence of an idempotent	23
7.	7. Properly nilpotent quantities	24
8.	8. The Peirce decomposition	24
9.	9. Principal idempotents	25
10.	10. Primitive idempotents	26
11.	11. Difference algebras	27
12.	12. Direct sums	28
13.	13. Reduction to irreducible components	29
14.	14. The centrum of a direct sum	30
15.	15. Scalar extensions of separable fields	31
16.	16. Inseparable fields	32
17.	17. Scalar extensions of the centrum	35
III. THE STRUCTURE THEOREMS OF WEDDERBURN		
1.	1. Semi-simple algebras	37
2.	2. Reduction to simple components	38
3.	3. Structure of simple algebras	39
4.	4. Direct products of normal algebras	41
5.	5. A fundamental property of normal simple algebras	41
6.	6. Normal simple algebras	42
7.	7. Separable algebras	44
8.	8. Structure of algebras with a radical	45
IV. SIMPLE ALGEBRAS		
1.	1. The uniqueness theorem	49
2.	2. Normal simple subalgebras as direct factors	51

3. Elementary properties	51
4. Subfields of a total matric algebra	52
5. Simple subalgebras	53
6. Extensions of equivalences	54
7. The existence of maximal subfields of normal division algebras	56
8. The class group	58
9. Index reduction factor	59
10. Representation of fields by normal simple algebras	60
11. Splitting fields of an algebra	61
12. Finite simple algebras	62
13. Applications of the Galois theory	62
v. CROSSED PRODUCTS AND EXPONENTS	
1. Connections of the theories	65
2. Equivalence of algebra-group pairs	65
3. Crossed products	66
4. Factor sets	67
5. Construction of crossed products	68
6. Direct products of crossed products	71
7. Scalar extensions of crossed products	72
8. Normalizations of crossed products	73
9. Elementary properties of cyclic algebras	74
10. The exponent of a normal simple algebra	75
vi. CYCLIC SEMI-FIELDS	
1. Groups of automorphisms of algebras	78
2. Notational hypotheses	79
3. Semi-fields	81
4. Diagonal direct factors	82
5. Cyclic semi-fields	83
6. Automorphisms of a direct product	85
7. Uniqueness of direct factorization	85
8. Direct products of cyclic semi-fields	86
9. Cyclic systems	87
10. The group of cyclic systems	89
11. Powers of cyclic systems	91
vii. CYCLIC ALGEBRAS AND p-ALGEBRAS	
1. Generalized cyclic algebras	93
2. Elementary results	95
3. Applications of the theory of cyclic systems	95
4. Norms and exponents	97
5. Algebras of prime-power degree	99
6. Lemmas on pure inseparable fields	101
7. Elementary properties of p -algebras	104
8. p -algebras with simple, pure inseparable splitting fields	106
9. Similarity of p -algebras to direct products of cyclic p -algebras	108
viii. REPRESENTATIONS AND RIEMANN MATRICES	
1. Representations of algebras	110
2. Matric representations	111
3. Reducibility of representations	113
4. Enveloping algebras	113
5. Reduction to irreducible components	114
6. Decomposable representations	115
7. Irreducible representations	116

CHAPTER	PAGE
8. Fully decomposable representations	118
9. Irreducible components of arbitrary matrix representations	119
10. Scalar extensions	121
11. The characteristic and minimum functions	122
12. The discriminant matrix	124
13. Generalized Riemann matrices	125
ix. RATIONAL DIVISION ALGEBRAS	
1. Algebras over an algebraic number field	129
2. Integral domains of an algebra	129
3. The p -adic fields \mathfrak{K}_p	131
4. Arithmetic theory of division algebras over \mathfrak{K}_p	132
5. The Hensel Lemma	136
6. Division algebras over any p -adic field	136
7. Structure of fields of finite degree over a p -adic field	138
8. The automorphism group of an unramified field	141
9. p -adic normal simple algebras	142
10. Quaternion algebras	145
11. Simple algebras over an ordered closed field	146
12. Lemmas from the theory of algebraic numbers	147
13. The p -adic extensions of algebraic number fields	148
14. Determination of all rational division algebras	149
15. The equivalence of normal simple algebras over an algebraic number field	150
x. INVOLUTIONS OF ALGEBRAS	
1. Definition and elementary properties of involutions	151
2. The J -symmetric and J -skew quantities	151
3. The two types of involutions	153
4. Involutions over \mathfrak{S} of a simple algebra	154
5. Involutions of a direct product	155
6. The construction of involutions	157
7. J -symmetric subfields	157
8. Involutorial crossed products	158
9. Involutorial simple algebras of the first kind	160
10. Involutorial quaternion algebras of the second kind	161
11. Involutorial simple algebras over an algebraic number field	162
12. Total real and pure imaginary fields	163
13. Special subfields of multiplication algebras	166
14. The structure of multiplication algebras	167
15. Multiplication algebras over an algebraic number field	170
xi. SPECIAL RESULTS	
1. Remarks on the structure of arbitrary algebras	171
2. Division algebras over special fields	172
3. The exponent of a normal division algebra	174
4. Normal division algebras with a pure maximal subfield	175
5. The structure of normal division algebras of degree three	177
6. The structure of normal division algebras of degree four	179
7. The construction of crossed products	182
8. Literature on non-associative algebras	188
9. Riemann matrices	188
10. Supplementary reading	190
11. Bibliography	192