

# Contents

Preface	iii
I. VALUATION RINGS	1
1. Valuation rings	1
2. Totally ordered abelian groups	7
3. Valuations	10
4. Ideals	13
5. Maximal and almost maximal valuation rings	20
6. Prüfer domains	26
Notes (Problem 1)	30
II. PRELIMINARIES ON MODULES	31
1. Modules	31
2. Divisibility	35
3. Relative divisibility (RD)	39
4. Pure submodules	44
5. Lemmas on pure submodules	48
6. Cyclic purity	51
7. Modules with local endomorphism rings	53
Notes	55
III. HOMOLOGICAL PRELIMINARIES	57
1. Homological background	57
2. Lemmas on Hom and Ext	61
3. Lemmas on tensor and torsion products	65

<b>IV.</b>	<b>PROJECTIVITY AND PROJECTIVE DIMENSION</b>	<b>70</b>
1.	Projective and flat modules	70
2.	Projective dimension	72
3.	Projective dimensions of torsion-free modules	77
4.	Projective dimension one	82
5.	Tight systems	87
6.	Quasi-projectivity	90
	Notes (Problems 2, 3)	93
<b>V.</b>	<b>TOPOLOGY AND FILTRATIONS</b>	<b>94</b>
1.	The R-topology	94
2.	R-complete modules	102
3.	Filtration and ultracompleteness	107
4.	The annihilator filtration	110
5.	R-ultracomplete modules	112
	Notes (Problem 4)	115
<b>VI.</b>	<b>DIVISIBILITY AND INJECTIVITY</b>	<b>116</b>
1.	Divisible modules	116
2.	h-divisible modules	119
3.	Divisible modules of projective dimension one	123
4.	Injective modules	130
5.	The injective dimension	134
6.	Quasi-injectivity	136
	Notes (Problems 5, 6)	139
<b>VII.</b>	<b>UNISERIAL MODULES</b>	<b>140</b>
1.	Uniserial modules	140
2.	Endomorphism rings of uniserial modules	144
3.	Non-standard uniserial modules	147
4.	Direct sums of uniserial modules	152
	Notes (Problem 7)	155
<b>VIII.</b>	<b>HEIGHTS AND INDICATORS</b>	<b>156</b>
1.	Heights	156
2.	Equiheight submodules	161
3.	Indicators	162
4.	Irregularities of indicators	164
5.	Smoothness	168
	Notes (Problem 8)	171
<b>IX.</b>	<b>FINITELY GENERATED AND POLYSERIAL MODULES</b>	<b>173</b>
1.	Finitely generated modules	174
2.	The Goldie dimension	178
3.	Indecomposable finitely generated modules	181
4.	Decompositions of finitely generated modules	185
5.	Polyserial modules	189
	Notes (Problems 9-17)	193

X.	INVARIANTS AND BASIC SUBMODULES	195
1.	$\alpha$ -Invariants	196
2.	$\alpha$ -Invariants of equiheight submodules	199
3.	$\alpha$ -Basic submodules	201
4.	Modules with trivial $\alpha$ -invariants	205
	Notes (Problems 18, 19)	208
XI.	RD-INJECTIVITY AND PURE-INJECTIVITY	209
1.	RD-injective modules	210
2.	Pure-injective modules	214
3.	Pure-injective modules over Prüfer domains	220
4.	Pure-injectivity over valuation domains	223
5.	Pure-injective hulls of polyserial modules	228
	Notes (Problems 20-23)	231
XII.	TORSION-COMPLETE AND COTORSION MODULES	233
1.	Torsion-complete modules	233
2.	Torsion-ultracomplete modules	238
3.	Cotorsion modules	242
4.	The cotorsion hull	245
	Notes (Problem 24)	248
XIII.	TORSION MODULES	250
1.	Embedding in pure polyserial submodules	251
2.	Separable modules	253
3.	Submodules of separable modules	256
4.	Direct sums of cyclic modules	259
5.	Torsion modules of projective dimension one	262
6.	Modules with zero $\alpha$ -invariants	264
	Notes (Problem 25)	268
XIV.	TORSION-FREE MODULES	269
1.	Preliminaries	269
2.	Completely decomposable modules	273
3.	Finite rank modules over almost maximal valuation domains	277
4.	Rank one dense basic submodules	281
5.	Chains of pure submodules	286
6.	Pure submodules of free modules	292
7.	Slender modules	294
	Notes (Problem 26)	301
	References	303
	Notation	309
	Author Index	312
	Subject Index	314