

Contents

Conventions and Notations	viii
Introduction	ix
1 Background material from algebraic topology	1
1.1 Spaces of maps	1
1.2 Homotopy groups	2
1.3 The Hurewicz theorem	9
1.4 The Whitehead theorem	11
1.5 CW-complexes and cellular homology	13
1.6 Fibrations and fibre bundles	17
1.7 Paracompact spaces	23
1.8 Simplicial sets	24
1.9 The Milnor exact sequence	29
2 Cohomology of groups	31
2.1 Overview of group cohomology	31
2.2 Eilenberg–MacLane spaces	34
2.3 Principal G -bundles	37
2.4 Classifying spaces	40
2.5 K-theory	46
2.6 Characteristic classes	51
2.7 Transfer	53
2.8 Stable cohomotopy and the Segal conjecture	59
2.9 Cohomology of general linear groups	63
2.10 The plus construction and algebraic K-theory	70
2.11 Hochschild homology	75
2.12 Free loops on BG	79
2.13 Cyclic homology	82
2.14 Cyclic sets	87
2.15 Extended centralisers	93
3 Spectral sequences	95
3.1 Introduction to spectral sequences	95
3.2 The spectral sequence of a filtered chain complex	100

3.3	The spectral sequence of a fibration	107
3.4	The spectral sequence of a double complex	109
3.5	The spectral sequence of a group extension	112
3.6	The Künneth spectral sequence	114
3.7	The Eilenberg–Moore spectral sequence	116
3.8	The Atiyah spectral sequence	118
3.9	Products in spectral sequences	119
3.10	Equivariant cohomology and finite generation	121
4	The Evens norm map and the Steenrod algebra	125
4.1	The Evens norm map	125
4.2	Finite generation of cohomology	130
4.3	The Bockstein homomorphism	136
4.4	Steenrod operations	140
4.5	Proof of the properties	143
4.6	Adem relations	149
4.7	Serre’s theorem on products of Bocksteins	152
4.8	Steenrod operations and spectral sequences	155
5	Varieties for modules and multiple complexes	157
5.1	Overview and historical background	157
5.2	Restriction to elementary abelian subgroups	159
5.3	Poincaré series and complexity	161
5.4	Varieties and commutative algebra	166
5.5	Example: extraspecial 2-groups	174
5.6	The Quillen stratification	178
5.7	Varieties for modules	181
5.8	Rank varieties	186
5.9	The modules L_ζ	190
5.10	Periodic modules	195
5.11	Andrews’ theorem	196
5.12	The variety of an indecomposable kG -module is connected	198
5.13	Example: dihedral 2-groups	199
5.14	Multiple complexes	203
5.15	Gaps in group cohomology	210
5.16	Isomorphisms in group cohomology	212
5.17	Poincaré duality	213
5.18	Cohen–Macaulay cohomology rings	216
6	Group actions and the Steinberg module	219
6.1	G -simplicial complexes	219
6.2	G -posets	221
6.3	The Lefschetz Invariant	223
6.4	Equivariant homotopy	224

6.5	Quillen's lemma	227
6.6	Equivalences of subgroup complexes	229
6.7	The generalised Steinberg module	231
6.8	Chevalley groups: a crash course	233
6.9	Steinberg module inversion and Alperin's conjecture	238
7	Local coefficients on subgroup complexes	241
7.1	Local coefficients	241
7.2	Constructions on coefficient systems	243
7.3	Chain complexes and homology of coefficient systems	246
7.4	Symplectic and orthogonal groups	248
7.5	Smith's theorem and universal coefficient systems	251
	Bibliography	257
	Index	273