

Contents

Preface	xi
I The Brauer group of a commutative ring	1
1 Morita theory for algebras without a unit	3
1.1 Morita contexts	3
1.2 Dual pairs and elementary algebras	10
1.3 Invertible modules	13
1.4 Left modules versus bimodules	15
2 Azumaya algebras and Taylor-Azumaya algebras	19
2.1 Central algebras, the separator and the trace map	19
2.2 Taylor-Azumaya algebras	23
2.3 The Rosenberg-Zelinsky exact sequence	33
3 The Brauer group	35
3.1 Equivalent Taylor-Azumaya algebra	35
3.2 The (big) Brauer group	39
3.3 The splitting theorem for Taylor-Azumaya algebras	41
3.4 The determinant map for an Azumaya algebra	48
3.5 The splitting theorem for semilocal rings	49
4 Central separable algebras	55
4.1 Separable algebras	55
4.2 Central separable algebras	60
4.3 Flat Taylor-Azumaya algebras	70
5 Amitsur cohomology and étale cohomology	77
5.1 Grothendieck topologies	77

5.2	Amitsur cohomology	79
5.3	The category of sheaves	81
5.4	Direct and inverse image sheaves and presheaves	87
5.5	Stalks in the étale topology	91
5.6	Etale cohomology	96
5.7	Flabby sheaves	102
6	Cohomological interpretation of the Brauer group	107
6.1	Cohomology with values in the category of invertible modules	107
6.2	The Brauer group versus the second cohomology group	113
6.3	Taylor's theorem	121
6.4	Verschoren's construction and Takeuchi's exact sequence	130
6.5	The Brauer group is torsion	134
6.6	The Mayer-Vietoris exact sequence	136
6.7	Gabber's theorem	139
6.8	The Brauer group modulo a nilpotent ideal	142
6.9	The Brauer group of a regular ring	144
6.10	Further results and examples	149
6.11	The Brauer group of a scheme and further generalizations	158
II	Hopf algebras and Galois theory	171
7	Hopf algebras	173
7.1	Algebras, coalgebras and Hopf algebras	173
7.2	Modules and comodules	189
8	Galois objects	197
8.1	Relative Hopf modules and Galois objects	197
8.2	Galois objects and graded ring theory	205
8.3	Galois objects and Morita theory	207
8.4	Galois extensions	210
8.5	Galois objects and classical Galois theory	212
8.6	Integrals	213

8.7 Galois coobjects	215
9 Cohomology over Hopf algebras	223
9.1 Sweedler cohomology	223
9.2 Harrison cohomology	226
10 The group of Galois (co)objects	235
10.1 Galois coobjects and Harrison cohomology	235
10.2 Galois coobjects with geometric normal basis	242
10.3 The group of Galois coobjects and Amitsur cohomology	247
10.4 The Picard group of a coalgebra	250
10.5 The group of Galois objects	261
10.6 The split part of the group of Galois objects	275
10.7 About the Picard invariant map	276
10.8 Pairings and noncommutative Galois objects	277
11 Some examples	283
11.1 Group algebras	283
11.2 Monogenic Larson orders	286
11.3 Examples in characteristic p	295
III The Brauer-Long group of a commutative ring	303
12 H-Azumaya algebras	305
12.1 Dimodules and dimodule algebras	305
12.2 H -Azumaya algebras	313
12.3 Separability conditions	317
12.4 Examples of H -Azumaya algebras	327
13 The Brauer-Long group of a commutative ring	339
13.1 The Brauer-Long group and its subgroups	339
13.2 The Brauer group of H -module Azumaya algebras	345
13.3 The Picard group of H -dimodules	351
13.4 The cup product	355

13.5 The split part of the Brauer-Long group	359
13.6 A bimodule version of the Rosenberg-Zelinsky exact sequence	366
13.7 A complex for the Brauer-Long group	368
13.8 The Hopf algebra $\mathcal{H} = \text{Hom}_R(H, K)$	372
13.9 A short exact sequence for the Brauer-Long group	375
13.10 Application to some particular cases	384
13.11 Computing $O(R, H)_{\min}$	405
13.12 The multiplication rules	412
13.13 The Brauer-Long group of a scheme	436
14 The Brauer group of Yetter-Drinfel'd module algebras	439
14.1 Yetter-Drinfel'd modules	439
14.2 H -Azumaya algebras and the Brauer group	442
14.3 The subgroups of $BQ(k, H)$	444
A Abelian categories and homological algebra	451
A.1 Abelian categories	451
A.2 Derived functors	453
B Faithfully flat descent	459
C Elementary algebraic K-theory	467
Bibliography	473
Index	480