

Contents

Symbols	xxiii
Preface	xxv
Acknowledgements	xxxix

Part I. An Array of Twentieth Century Associative Algebra

Chapter 1. Direct Product and Sums of Rings and Modules and the Structure of Fields	1
§1.1 General Concepts	1
§1.2 Internal Direct Sums	2
§1.3 Products of Rings and Central Idempotents	3
§1.4 Direct Summands and Independent Submodules	3
§1.5 Dual Modules and Torsionless Modules	3
§1.6 Torsion Abelian Groups	4
§1.7 Primary Groups	4
§1.8 Bounded Order	4
§1.9 Theorems of Zippin and Frobenius-Stickelberger	4
§1.10 Divisible Groups	5
§1.11 Splitting Theorem for Divisible Groups	5
§1.12 Second Splitting Theorem	5
§1.13 Decomposition Theorem for Division Groups	5
§1.14 Torsion Group Splits Off Theorem	5
§1.15 Fundamental Theorem of Abelian Groups	5
§1.15 Kulikoff's Subgroup Theorem	5
§1.16 Corner's Theorem and the Dugas-Göbel Theorem	6
§1.17 Direct Products as Summands of Direct Sums	6
§1.18 Baer's Theorem	6
§1.19 Specker-Nöbeling-Balcerzyk Theorems	6
§1.20 Dubois' Theorem	7
§1.21 Balcerzyk, Białynicki, Birula and Łoś Theorem, Nunke's Theorem, and O'Neill's Theorem	7
§1.22 Direct Sums as Summands of Their Direct Product	8
§1.23 Camillo's Theorem	8
§1.24 Lenzing's Theorem	8
§1.25 Zimmermann's Theorem on Pure Injective Modules	8

§1.26	Szele-Fuchs-Ayoub-Huynh Theorems	8
§1.27	Kertész-Huynh-Tominaga Torsion Splitting Theorems	9
§1.28	Three Theorems of Steinitz on the Structure of Fields	9
§1.29	Lüroth's Theorem	11
§1.30	Artin-Schreier Theory of Formally Real Fields	11
§1.31	Theorem of Castelnuovo-Zariski	12
§1.32	Monotone Minimal Generator Functions	12
§1.33	Quigley's Theorem: Maximal Subfields without α	13
Chapter 2.	Introduction to Ring Theory: Schur's Lemma and Semisimple Rings, Prime and Primitive Rings, Noetherian and Artinian Modules, Nil, Prime and Jacobson Radicals	15
•	Quaternions	15
•	Hilbert's Division Algebra	16
•	When Everybody Splits	16
•	Artinian Rings and the Hopkins-Levitzki Theorem	17
•	Automorphisms of Simple Algebras: The Theorem of Skolem-Noether	18
•	Wedderburn Theory of Simple Algebras	19
•	Crossed Products and Factor Sets	19
•	Primitive Rings	20
•	Nil Ideals and the Jacobson Radical	20
•	The Chevalley-Jacobson Density Theorem	20
•	Semiprimitive Rings	21
•	Semiprimitive Polynomial Rings	21
•	Matrix Algebraic Algebras	21
•	Primitive Polynomial Rings	22
•	The Structure of Division Algebras	23
•	Ts'en's Theorem	23
•	Cartan-Jacobson Galois Theory of Division Rings	23
•	Historical Note: Artin's Question	24
•	Jacobson's $a^{n(a)} = a$ Theorems and Kaplansky's Generalization	24
•	Kaplansky's Characterization of Radical Field Extensions	25
•	Radical Extensions of Rings	25
•	The Cartan-Brauer-Hua Theorem on Conjugates in Division Rings	26
•	Hua's Identity	27
•	Amitsur's Theorem and Conjugates in Simple Rings	28
•	Invariant Subrings of Matrix Rings	28
•	Rings Generated by Units	29
•	Transvections and Invariance	30
•	Other Commutativity Theorems	30
•	Noetherian and Artinian Modules	31
•	The Maximum and Minimum Conditions	31
•	Inductive Sets and Zorn's Lemma	31
•	Subdirectly Irreducible Modules: Birkhoff's Theorem	31
•	Jordan-Hölder Theorem for Composition Series	33
•	Two Noether Theorems	33
•	Hilbert Basis Theorem	34
•	Hilbert's Fourteenth Problem: Nagata's Solution	35

• Noether's Problem in Galois Theory: Swan's Solution	35
• Realizing Groups as Galois Groups	35
• Prime Rings and Ideals	36
• Chains of Prime Ideals	36
• The Principal Ideal Theorems and the DCC on Prime Ideals	36
• Primary and Radical Ideals	37
• Lasker-Noether Decomposition Theorem	37
• Hilbert Nullstellensatz	38
• Prime Radical	39
• Nil and Nilpotent Ideals	40
• Nil Radicals	41
• Semiprime Ideals and Unions of Prime Ideals	42
• Maximal Annihilator Ideals Are Prime	43
• Rings with Acc on Annihilator Ideals	43
• The Baer Lower Nil Radical	44
• Group Algebras over Formally Real Fields	45
• Jacobson's Conjecture for Group Algebras	46
• Simplicity of the Lie and Jordan Rings of Associative Rings: Herstein's Theorems	46
• Simple Rings with Involution	47
• Symmetric Elements Satisfying Polynomial Identities	48
• Historical Notes	48
• Separable Fields and Algebras	49
• Wedderburn's Principal or Factor Theorem	49
• Invariant Wedderburn Factors	49
Chapter 3. Direct Decompositions of Projective and Injective Modules	51
• Direct Sums of Countably Generated Modules	51
• Injective Modules and the Injective Hull	52
• Injective Hulls: Baer's and Eckmann-Schöpf's Theorems	52
• Complement Submodules and Maximal Essential Extensions	52
• The Cantor-Bernstein Theorem for Injectives	53
• Generators and Cogenerators of Mod-R	53
• Minimal Cogenerators	54
• Cartan-Eilenberg, Bass, and Matlis-Papp Theorems	54
• Two Theorems of Chase	54
• Sets vs. Classes of Modules: The Faith-Walker Theorems	55
• Polynomial Rings over Self-injective or QF Rings	56
• Σ -injective Modules	57
• Quasi-injective Modules and the Johnson-Wong Theorem	57
• Dense Rings of Linear Transformations and Primitive Rings Revisited	58
• Harada-Ishii Double Annihilator Theorem	58
• Double Annihilator Conditions for Cogenerators	59
• Koehler's and Boyle's Theorems	59
• Quasi-injective Hulls	60
• The Teply-Miller Theorem	60
• Semilocal and Semiprimary Rings	60

• Regular Elements and Ore Rings	61
• Finite Goldie Dimension	62
• Cailleau's Theorem	62
• Local Rings and Chain Rings	62
• Uniform Submodules and Maximal Complements	63
• Beck's Theorems	63
• Dade's Theorem	64
• When Cyclic Modules Are Injective	65
• When Simple Modules Are Injective: V-Rings	65
• Cozzens' V-Domains	66
• Projective Modules over Local or Semilocal Rings, or Semihereditary Rings	66
• Serre's Conjecture, the Quillen-Suslin Solution and Seshadri's Theorem	67
• Bass' Theorem on When Big Projectives Are Free	67
• Projective Modules over Semiperfect Rings	67
• Bass' Perfect Rings	68
• Theorems of Björk and Jonah	69
• Max Ring Theorems of Hamsher, Koifman, and Renault	69
• The Socle Series of a Module and Loewy Modules	69
• Semi-Artinian Rings and Modules	70
• The Perlis Radical and the Jacobson Radical	70
• The Frattini Subgroup of a Group	71
• Krull's Intersection Theorem and Jacobson's Conjecture	71
• Nakayama's Lemma	71
• The Jacobson Radical and Jacobson-Hilbert Rings	72
• When Nil Implies Nilpotency	73
• Shock's Theorem	73
• Kurosch's Problem	74
• The Nagata-Higman Theorem	74
• \aleph_0 -Categorical Nil Rings Are Nilpotent	74
• The Golod-Shafarevitch Theorem	75
• Some Amitsur Theorems on the Jacobson Radical	75
• Köethe's Radical and Conjecture	76
• A General Wedderburn Theorem	76
• Koh's Schur Lemma	77
• Categories	77
• Morita's Theorem	77
• Theorems of Camillo and Stephenson	78
• The Basic Ring and Module of a Semiperfect Ring	78
• The Regularity Condition and Small's Theorem	79
• Reduced Rank	79
• Finitely Embedded Rings and Modules: Theorems of Vámos and Beachy	80
• The Endomorphism Ring of Noetherian and Artinian Modules	81
• Fitting's Lemma	81
• Köthe-Levitzki Theorem	82
• Levitzki-Fitting Theorem	83
• Kolchin's Theorem	84
• Historical Notes on Local and Semilocal Rings	85

• Further Notes for Chapter 3	87
• Free Subgroups of $GL(n, F)$	87
• Sanov's Theorem	87
• Hartley-Pickel Theorem	88
• Steinitz Rings	88
• Free Direct Summands	88
• Essentially Nilpotent Ideals	88
Chapter 4. Direct Product Decompositions of von Neumann Regular Rings and Self-injective Rings	89
• Flat Modules	90
• Character Modules and the Bourbaki-Lambek Theorem	90
• When Everybody Is Flat	91
• Singular Splitting	92
• Utumi's Theorems	93
• Weak or $F \cdot G$ Injectivity	93
• Abelian VNR Rings	94
• The Maximal Regular Ideal	94
• Products of Matrix Rings over Abelian VNR Rings	95
• Products of Full Linear Rings	95
• Dedekind Finite	95
• Jacobson's Theorem	96
• Shepherdson's and Montgomery's Examples	96
• Group Algebras in Characteristic 0 Are Dedekind Finite	96
• Prime Right Self-injective VNR Rings	97
• Goodearl-Handelman Characterization of Purely Infinite Rings	97
• Kaplansky's Direct Product Decompositions of VNR Rings	97
• Kaplansky's Conjecture on VNR Rings: Domanov's Counterexample and Goodearl's and Fisher-Snider's Theorems	98
• Azumaya Algebras	98
• Hochschild's Theorem on Separable Algebras	99
• The Auslander-Goldman-Brauer Group of a Commutative Ring	100
• Menal's Theorem on Tensor Products of SI or VNR Algebras	100
• Lawrence's Theorem on Tensor Products of Semilocal Algebras	100
• Armendariz-Steinberg Theorem	101
• Strongly Regular Extensions of Rings	101
• Pseudo-Frobenius (PF) Rings	101
• Kasch Rings	102
• FPF Rings	103
Chapter 5. Direct Sums of Cyclic Modules	105
• Uniserial and Serial Rings	105
• Nonsingular Rings	107
• FGC Rings	107
• Linearly and Semicompact Modules	108
• Maximal Rings	108
• Almost Maximal Valuation, and Arithmetic Rings	108
• Torch Rings	109

• Fractionally Self-injective Rings	109
• FGC Classification Theorem	110
• Maximal Completions of Valuation Rings	111
• MacLane's and Vámos' Theorems	111
• Gill's Theorem	111
• Vamosian Rings	112
• Quotient Finite Dimensional Modules	112
• The Genus of a Module and Generic Families of Rings	113
• The Product Theorem	114
• Serre's Condition	115
• FPF Split Null Extensions	116
• Characterization of Commutative FPF Rings	117
• Semiperfect FPF Rings	117
• Faticoni's Theorem	118
• Kaplansky's and Levy's Maximal Valuation Rings	118
• Page's Theorems	118
• Further Examples of Valuation Rings and PF Rings	119
• Historical Note	120
Chapter 6. When Injectives Are Flat: Coherent FP-injective Rings	121
• Pure Injective Modules	121
• Elementary Divisor Rings	123
• Stable Range and the Cancellation Property	124
• Fractionally Self FP-Injective Rings	124
• Coherent Rings: Theorems of Chase, Matlis and Couchot	125
• When Injective Modules Are Flat: IF Rings	126
• Power Series over VNR and Linear Compact Rings	126
• Historical Note	127
• Locally Split Submodules	127
• Existentially Closed Rings	128
• Existentially Closed Fields	129
• Other Embeddings in Skew Fields	129
• Galois Subrings of Ore Domains Are Ore	130
• Rings with Zero Intersection Property on Annihilators: Zip Rings	130
• On a Question of Mal'cev: Klein's Theorem	131
• Weakly Injective Modules	132
• Gauss Elimination and Weakly Injectivity	132
• Zip McCoy Rings	132
• Elementary Equivalence	134
• Pure-Injective Envelopes	135
• Ziegler's Theorem	136
• Noetherian Pure-Injective Rings	137
• Σ -Pure-Injective Modules	137
• Pure-Semisimple Rings	137
Chapter 7. Direct Decompositions and Dual Generalizations of Noetherian Rings	139
• PP Rings and Finitely Generated Flat Ideals	139

• Right Bezout Rings	140
• Faith-Utumi Theorem	140
• Finitely Embedded Rings	141
• Simple Noetherian Rings	141
• Simple Differential Polynomial Rings	142
• The Weyl Algebra	143
• When Modules Are Direct Sums of a Projective and a Noetherian Module	144
• When Modules Are Direct Sums of an Injective and a Noetherian Module	144
• Dual Generalizations of Artinian and Noetherian Modules	145
• Completely Σ -Injective Modules	146
• Ore Rings Revisited	147
• On Hereditary Rings and Boyle's Conjecture	148
• Δ -Injective Modules	150
• Co-Noetherian Rings	152
Chapter 8. Completely Decomposable Modules and the Krull-Schmidt-Azumaya Theorem	153
• Herbera-Shamsuddin and Camps-Dicks Theorems	153
• Swan's Theorem	154
• Evans' Theorem	154
• Matlis' Problem	154
• The Exchange Property and Direct Sums of Indecomposable Injective Modules	155
• Crawley-Jónsson Theorem	155
• Warfield, Nicholson and Monk Theorems	156
• Π -Regular Rings	157
• Yamagata's Theorem	158
• Decompositions Complementing Direct Summands	158
• Fitting's Lemma and the Krull-Schmidt Theorem	159
• A Very General Schur Lemma	160
• Rings of Finite and Bounded Module Type	160
Chapter 9. Polynomial Rings over Vamosian and Kerr Rings, Valuation Rings and Prüfer Rings	163
• Kerr Rings and the Camillo-Guralnick Theorem	163
• Rings with Few Zero Divisors Are Those with Semilocal Quotient Rings	164
• Manis Valuation Rings	165
• Integrally Closed Rings	165
• Kaplansky's Question	166
• Local Manis Valuation Rings	166
• Domination of Local Rings	167
• Marot Rings	168
• Krull Rings	169
• Rings with Krull	169
• Annie Page's Theorem	169

• The Maximal Quotient Ring of a Commutative Ring	169
• The Ring of Finite Fractions	170
• Prüfer Rings and Davis, Griffin and Eggert Theorems	170
• Strong Prüfer Rings	171
• Discrete Prüfer Domains	171
• Strongly Discrete Domains	171
• Generalized Dedekind Rings	172
• Facchini's Theorems on Piecewise Noetherian Rings	172
 Chapter 10. Isomorphic Polynomial Rings and Matrix Rings	 173
• Hochster's Example of a Non-unique Coefficient Ring	173
• Brewer-Rutter Theorems	173
• The Theorems of Abhyankar, Heinzer and Eakin	173
• Isomorphic Matrix Rings	174
• Lam's Survey	174
 Chapter 11. Group Rings and Maschke's Theorem Revisited	 175
• Connell's Theorems on Self-injective Group Rings	175
• Perfect and Semilocal Group Rings	176
• von Neumann Regular Group Rings	176
• Jacobson's Problem on Group Algebras	176
• Isomorphism of Group Algebras: The Perlis-Walker Theorem	176
• Dade's Examples	176
• Higman's Problem	177
• Theorems of Higman, Kasch-Kupisch-Kneser on Group Rings of Finite Module Type	177
• Janusz and Srinivasan Theorems	177
• Morita's Theorem	177
• Roseblade's Theorems on Polycyclic-by-Finite Group Rings	178
• A Weak Nullstellensatz	178
• Hilbert Group Rings	178
 Chapter 12. Maximal Quotient Rings	 179
• The Maximal Quotient Ring	180
• When $Q_{\max}^r(R) = Q_{\max}^{\ell}(R)$: Utumi's Theorem	181
• Courter's Theorem on When All Modules Are Rationally Complete	182
• Snider's Theorem on Group Algebras of Characteristic 0	182
• Galois Subrings of Quotient Rings	183
• Localizing Categories and Torsion Theories	184
• Ring Epimorphisms and Localizations	185
• Continuous Regular Rings	185
• Complemented and Modular Lattices	186
• von Neumann's Coordinatization Theorem	186
• von Neumann's Dimension Function	186
• Utumi's Characterization of Continuous VNR Rings	187
• Semi-continuous Rings and Modules	187
• CS Projective Modules	190
• Strongly Prime Rings	190

Chapter 13. Morita Duality and Dual Rings	193
• Dual Rings	196
• Skornyakov's Theorem on Self-dual Lattices of Submodules	196
• Hajarnavis-Norton Theorem	196
• Faith-Menal Theorem	197
• Commutative Rings with QF Quotient Rings	198
• On a Vasconcelos Conjecture	198
• Kasch-Mueller Quasi-Frobenius Extensions	199
• Balanced Rings and a Problem of Thrall	199
• When Finitely Generated Modules Embed in Free Modules	200
• A Theorem of Pardo-Asensio and a Conjecture of Menal	202
• Johns' Rings Revisited	202
• Two Theorems of Gentile and Levy on When Torsionfree Modules Embed in Free Modules	203
• When an Ore Ring Has Quasi-Frobenius Quotient Ring	203
• Levy's Theorem	203
Chapter 14. Krull and Global Dimensions	205
• Homological Dimension of Rings and Modules	206
• The Hilbert Syzygy Theorem	207
• Regular Local Rings	209
• Noncommutative Rings of Finite Global Dimension	210
• Classical Krull Dimension	211
• Krull Dimension of a Module and Ring	211
• Critical Submodules	214
• Acc on Radical Ideals (Noetherian Spec)	215
• Goodearl-Zimmermann-Huisgen Upper Bounds on Krull Dimension	215
• McConnell's Theorem on the n -th Weyl Algebra	218
• Historical Note	218
Chapter 15. Polynomials Identities and PI-Rings	221
• Amitsur-Levitski Theorem	223
• Kaplansky-Amitsur Theorem	223
• Posner's Theorem	224
• Nil PI-Algebras Are Locally Nilpotent	224
• Rowen PI-Algebras	225
• Generic Matrix Rings Are Ore Domains	226
• Generic Division Algebras Are Not Crossed Products	226
• When Fully Bounded Noetherian Algebras Are PI-Algebras	226
• Notes on Prime Ideals	227
• Historical Notes	227
Chapter 16. Unions of Primes, Prime Avoidance, Associated Prime Ideals, Acc on Irreducible Ideals, and Annihilator Ideals in Commutative Rings	229
• McCoy's Theorem	229
• The Baire Category Theorem and the Prime Avoidance Theorem	229
• W. W. Smith's Prime Avoidance Theorem and Gilmer's Dual	230
• Irreducible Modules Revisited	231

• (Subdirectly) Irreducible Submodules	231
• Associated Prime Ideals	233
• Chain Conditions on Annihilators	235
• Semilocal Kasch Quotient Rings	237
• Acc_\perp Rings Have Semilocal Kasch Quotient Rings	238
• Beck's Theorem	238
• Acc on Irreducible Right Ideals	239
• Nil Singular Ideals	239
• Primary Ideals	240
• Characterization of Noetherian Modules	241
• Camillo's Theorem	242

Chapter 17. Dedekind's Theorem on the Independence of Automorphisms Revisited

• Conventions	243
• Résumé of Results	244
• Dependent Automorphisms of Polynomial and Power Series Rings	244
• Normal Basis	245
• The Dependence Theorem	245
• The Skew Group Ring	246
• The Induction Theorem	247
• Radical Extensions	247
• Partial Converse to Theorem 17.4	248
• Kaplansky's Theorem Revisited	248
• Reduced Rings	249
• The Role of Ideals in Dependency	250
• Galois Subrings of Independent Automorphism Groups of Commutative Rings Are Quorite	250
• Automorphisms Induced in Residue Rings (For Sam Perlis on His 85th Birthday)	252
• Notes on Independence of Automorphisms	253
• Letter from Victor Camillo (Excerpts)	254

Part II. Snapshots

Chapter 18. Some Mathematical Friends and Places	255
• Some Profs at Kentucky and Purdue	255
• Mama and Sis	256
• Perlis' Pearls	256
• The Ring's the Thing	257
• My "Affair" with Ulla	257
• How I Taught Fred to Drive	257
• "The Old Dog Laughed To See Such Fun"	258
• My "Lineage"—Math and Other	258
• Big Brother—"Edgie"	258
• H.S.F. Jonah and C. T. Hazard	259
• John Dyer-Bennet and Gordon Walker	259
• Henriksen, Gilman, Jerison, McKnight, Kohls and Correl	260

• Joop and Vilna, Len and Reba	260
• Some Fellow Students at Purdue	260
• Michigan State University (1955–1957)	261
• Sam Berberian, Bob Blair, Gene Deskins, and the Oehmkes	261
• “Cupcake”	262
• Leroy M. Kelly, Fritz Herzog, Ed Silverman and Vern Grove	262
• Orrin Frink	262
• Gottfried Köthe and Fritz Kasch	263
• Romantische Heidelberg	263
• Reinhold Baer	263
• Death in Munich (1960)	264
• Marston Morse and the Invitation to the Institute	264
• What Frau Seifert Told Me	265
• “Some Like It Hot” (Manche Mög Es Heiss)	265
• Marston Morse	265
• Marston and Louise	266
• Louise Morse: Picketing IDA	266
• Kay and Deane Montgomery	267
• “Leray Who?”	267
• How Deane Helped Liberate Rutgers	268
• Hassler Whitney	268
• John Milnor	269
• Paul Fussell	269
• Hetty and Atle Selberg	269
• Another Invitation to the Institute	270
• The Idea of the Institute As an Intellectual Hotel	270
• Oppie and Kitty	270
• Gaby and Armand Borel	270
• Gaby	271
• Alliluyeva	271
• George F. Kennan	271
• Kennan’s Memoirs	272
• Kurt Gödel	273
• P. J. Cohen	273
• Kurosch Meets Witt	273
• Hitler’s View of the Institute	274
• The Interesting Case of Threlfall	274
• My Friendship with Witt	274
• My First Paper at the Institute: Communicated by Nathan Jacobson	275
• “Proofs Too Short”	275
• Caroline D. Underwood	275
• Mort and Karen Brown	275
• Leah and Clifford Spector	276
• John Ernest	276
• “I Like This Motel”	276
• Institute Cats	276
• Yitz	277
• Injective Modules and Quotient Rings	277
• Fritz, Bruno, Rudy, and Ulrich	277

• The High Cost of Living in Germany (1959–1960)	277
• Steve Chase	278
• The Institute and Flexner’s Idea	278
• Lunch with Dyson, Lee, Yang and Pais	278
• Helen Dukas	279
• Arthur and Dorothy Guy	279
• Patricia Kelsh Woolf	280
• Johnny von Neumann and “The Maniac”	281
• Who Got Einstein’s Office?	281
• The Walkers, Frank Anderson, and Eben Matlis	281
• Carol and Elbert	282
• “Waiting for Gottfried”	282
• Harish-Chandra	282
• Veblen, Tea, and the Arboretum	283
• “On the Banks of the Old Raritan” (School Song)	283
• The Bumby-Osofsky Theorem	284
• Osofsky’s Ph.D. Thesis	284
• Yuzo	284
• At the Stockholm ICM (1962)	285
• Nathan Jacobson	285
• How Jake Helped Me and Rutgers	286
• Vic, John, Midge, and Ann	286
• A Problem of Bass and Cozzens’ Ph.D. Thesis	287
• Boyle’s Ph.D. Thesis and Conjecture	287
• A Problem of Thrall and Camillo’s Ph.D. Thesis	287
• Avraham and Ahuva	288
• Abraham Zaks	288
• Professor Netanyahu	288
• Jonathan and Hembda Golan	288
• Shimshon Amitsur	288
• Amitsur’s “Absence of Leave”	289
• Miriam Cohen	289
• Joy Kinsburg	289
• Paul Erdős	290
• What Is Your Erdős Number?	290
• Piatetski-Shapiro Is Coming!	291
• Gerhard Hochschild on Erdős	291
• Joachim Lambek	291
• S. K. Jain and India	292
• Kashmiri Gate at 5:00 P.M.	292
• Toot-Toot for a Day! Toot-Toot for an Age!	293
• The Rupee Mountain	294
• K. G. Ramanathan and Bhama Srinivasan (Bombay and Madras)	294
• The Indian Idea of Karma	294
• Joan and Charles Neider	295
• Charley	295
• Louis Fischer and Gandhi	296
• Sputnik!	296
• Govaru Po Russki? My Algebra Speaks Russian	296

• Walter Kaufmann and Nietzsche	297
• Hessay and Earl Taft	297
• Kenneth Wolfson, Antoni Kosinski, and Glen Bredon	298
• Paul Moritz Cohn	299
• Joanne Elliott, Vince Cowling, and Jane Scanlon	299
• Rutgers Moves Up!	299
• Roz Wolfson	300
• The George William Hill Center	300
• Daniel Gorenstein and the Classification of Simple Groups	300
• The Monster Group	301
• Danny and Yitz	301
• Gorenstein Rings	301
• All the News That Is Fit To Print” - <i>New York Times</i>	301
• The Gorenstein Report and “Dream Time”	302
• Helen and Danny	302
• Ken Goodearl, Joe Johnson, and John Cozzens	303
• Hopkins and Levitzki	304
• Jakob Levitzki	304
• Chuck Weibel and Tony Geramita at the Institute (1977-1978)	304
• How I Helped Recruit Chuck	305
• Poobhalan Pillay, Lalita, and Karma	305
• “Tommy” Tominaga and “Tokyo Rose”	306
• Ted Faticoni, the Walkers and Me at Las Cruces	306
• New Mexico	307
• Rio Grande	307
• Dolores Herbera and Ahmad Shamsuddin at Rutgers (1993-1994)	308
• Pere Menal	308
• Alberto Facchini and More Karma	309
• Barcelona and Bellaterra	310
• Gaudi’s Genius	311
• The Ramblas	311
• Norman Steenrod	312
• Kaplansky, Steenrod and Borel	312
• Kap	312
• Kap’s “Rings and Things”	313
• “The World’s Greatest Algebra Seminar”	313
• Samuel Eilenberg	313
• Myles “Tiernovsky”	314
• Sammy Collects Indian Sculpture	314
• “The Only Thing They Would Let Us Do”	314
• Emil Artin	315
• Michael Artin	316
• University Towns	316
• Some Cafés and Coffee Houses	317
• “Crazy Eddie”, Svetlana, “Captain” Bill, and Jay	318
• Jay and Stan	318
• Roy Hutson and Vic Camillo—Two Poet Mathematicians	319
• Marc Rieffel, Serge Lang, Steve Smale and Me	319
• Parlez-Vous Français? My Proof Speaks French	320

• Mario Savio and The Berkeley Free Speech Movement (1964)	320
• Jerry Rubin	321
• Steve Smale	321
• Some Undergraduate Gems at Rutgers and Penn State	321
• “Carl, You Will Always Have Dumb Students”	322
• Envoi to My Century	323
Index to Part II	S-1
Bibliography	325
Register of Names	387
Index of Terms and Authors of Theorems	395