

CONTENTS

Preface	vii
1. Basic concepts	1
1.1. Lie algebras	1
1.2. Subalgebras, ideals and quotient algebras	4
1.3. Simple algebras	7
1.4. Direct sum	11
1.5. Derived series and descending central series	12
1.6. Killing form	16
2. Nilpotent and solvable Lie algebras	21
2.1. Preliminaries	21
2.2. Engel's theorem	22
2.3. Lie's theorem	24
2.4. Nilpotent linear Lie algebras	26
3. Cartan subalgebras	31
3.1. Cartan subalgebras	31
3.2. Existence of Cartan subalgebras	34
3.3. Preliminaries	36
3.4. Conjugacy of Cartan subalgebras	41
4. Cartan's criterion	44
4.1. Preliminaries	44
4.2. Cartan's criterion for solvable Lie algebras	45
4.3. Cartan's criterion for semisimple Lie algebras	47
5. Cartan decompositions and root systems of semisimple Lie algebras	48
5.1. Cartan decompositions of semisimple Lie algebras	48
5.2. Root systems of semisimple Lie algebras	53
5.3. Dependence of structure of semisimple Lie algebras on root systems	58
5.4. Root systems of the classical Lie algebras	66
6. Fundamental systems of roots of semisimple Lie algebras and Weyl groups	74
6.1. Fundamental systems of roots and prime roots	74
6.2. Fundamental systems of roots of the classical Lie algebras	80
6.3. Weyl groups	82
6.4. Properties of Weyl groups	86
7. Classification of simple Lie algebras	92
7.1. Diagrams of π systems	92
7.2. Classification of simple π systems	93
7.3. The Lie algebra G_2	100
7.4. Classification of simple Lie algebras	102

8. Automorphisms of semisimple Lie algebras	105
8.1. The group of automorphisms and the derivation algebra of a Lie algebra	105
8.2. The group of outer automorphisms of a semisimple Lie algebra	108
9. Representations of Lie algebras	116
9.1. Fundamental concepts	116
9.2. Schur's lemma	119
9.3. Representations of the three-dimensional simple Lie algebra	120
10. Representations of semisimple Lie algebras	126
10.1. Irreducible representations of semisimple Lie algebras	126
10.2. Theorem of complete reducibility	134
10.3. Fundamental representations of semisimple Lie algebras	142
10.4. Tensor representations	145
10.5. Elementary representations of simple Lie algebras	148
11. Representations of the classical Lie algebras	151
11.1. Representations of A_n	151
11.2. Representations of C_n	155
11.3. Representations of B_n	156
11.4. Representations of D_n	158
12. Spin representations and the exceptional Lie algebras	160
12.1. Associative algebras	160
12.2. Clifford algebra	161
12.3. Spin representations	165
12.4. The exceptional Lie algebras F_4 and E_8	168
13. Poincaré–Birkhoff–Witt theorem and its applications to representation theory of semisimple Lie algebras	180
13.1. Enveloping algebras of Lie algebras	180
13.2. Poincaré–Birkhoff–Witt theorem	182
13.3. Applications to representations of semisimple Lie algebras	188
14. Characters of irreducible representations of semisimple Lie algebras	190
14.1. A recursion formula for the multiplicity of a weight of an irreducible representation	190
14.2. Half of the sum of all the positive roots	197
14.3. Alternating functions	200
14.4. Formula of the character of an irreducible representation	203
15. Real forms of complex semisimple Lie algebras	210
15.1. Complex extension of real Lie algebras and real forms of complex Lie algebras	210
15.2. Compact Lie algebras	212
15.3. Compact real forms of complex semisimple Lie algebras	215
15.4. Roots and weights of compact semisimple Lie algebras	221
15.5. Real forms of complex semisimple Lie algebras	223
Index	227
Other Titles in the Series in Pure and Applied Mathematics	229