

CONTENTS

VOLUME ONE

CHAPTER I. Elementary Concepts of Lie Algebras

§.1.1. Lie Algebras	1
§.1.2. Some Properties of Lie Composition	11
§.1.3. Lie-algebra Homomorphisms and Exact Sequences	17
§.1.4. Derivation Mappings (i.e. "derivation")	19
§.1.5. Solvability	35
§.1.6. Nilpotency	55
§.1.7. Radical, Nil-radical, Simplicity and Semi- simplicity	72

CHAPTER II. Some General Properties of Lie Algebra

§.2.1. Lie-Algebra Direct Sums	79
§.2.2. Lie-Algebra Semi-direct Sum	90
§.2.3. Representations and Lie Modules	95
§.2.4. Extensions of Lie Algebras	113
§.2.5. Bilinear Forms on Lie Algebra	145

CHAPTER III. Universal Enveloping Algebra and Associa-
tively-generated Algebra

§.3.1. The Universal Enveloping Algebra of a Lie Algebra	161
§.3.2. Explicit Construction of a Universal Enveloping Algebra	169

§.3.3. On the Graded Structure of a Universal Enveloping Algebra	178
§.3.4. Poincare-Birkhoff-Witt Theorem	186
§.3.5. The Associatively-generated Algebra for a Linear Lie Algebra	228

**CHAPTER IV. Some Fundamental Theorems on Solvability,
Nilpotency and Semi-simplicity**

§.4.1. Lie's Theorem on Solvable Lie Algebras . . .	237
§.4.2. Variations on a Theme of Lie's Theorem . . .	251
§.4.3. Engel's Theorem on Nilpotent Lie Algebras . .	267
§.4.4. Variations on a Theme of Engel's Theorem . .	280
§.4.5. Cartan's Criterion for Solvability and Semi-Simplicity	287
§.4.6. Some Properties of Radicals	320

CHAPTER V. Further General Theorems on Semi-simplicity

§.5.1. Concept of Orthogonality w.r.t. a Bilinear Form	345
§.5.2. Casimir Elements and Casimir Operators . . .	361
§.5.3. Weyl's Theorem on Complete Reducibility . .	374
§.5.4. Extensions of Semi-simple Lie Algebras . . .	415
§.5.5. Levi Decomposition and Malcev-H. Chandra's Theorem	437

CHAPTER VI. Cartan Subalgebras, Weights and Roots

§.6.1. Fitting Decomposition of a Lie Algebra	463
§.6.2. Cartan Subalgebras and Rank of a Lie Algebra	472
§.6.3. Weights and Roots	482

CHAPTER VII. Structure of Split Semi-simple Lie Algebra

§.7.1. Some Properties of Roots and Root-modules	510
§.7.2. Series and Strings of Roots	532
§.7.3. Positive and Fundamental Roots	543
§.7.4. Cartan Matrices and Their Properties	564
§.7.5. Dynkin Diagrams and Classification of Split Simple Lie Algebras	585
§.7.6. Explicit Construction of Split Simple Classical Lie Algebras	608
§.7.7. Explicit Construction of Split Simple Exceptional Lie Algebras	644

CHAPTER VIII. On Conjugacy of Cartan Subalgebras

§.8.1. Notions on Polynomial Mappings	691
§.8.2. On Some Properties of a Regular Element	706

CHAPTER IX. Representations of Semi-simple Lie Algebras

§.9.1. Preliminaries	723
§.9.2. Dominant Weight	730
§.9.3. Simple Lie Modules	749

CHAPTER X. Some Cohomological and Functorial Properties
of Lie Algebras

§.10.1.* Elementary Definitions in Category Theory;	
Categories and Functors	783
§.10.2.* Zero-objects, Pull-backs, Kernels and	
Abelian Categories	796
§.10.3.* Functorial Properties of K-modules	811
§.10.4.* Functorial Properties of K-module	
Extensions	824
§.10.5.* Homology and Cohomology Groups and	
Derived Functors	841
§.10.6. Cohomology Theory of a Lie Algebra	856
 BIBLIOGRAPHY	
INDEX	887
	890