

CONTENTS

| | |
|--|----|
| Preface | v |
| Lecture 1 | 1 |
| 1.1. The Lie algebra d of complex vector fields on the circle | 1 |
| 1.2. Representations $V_{\alpha, \beta}$ of d | 4 |
| 1.3. Central extensions of d : the Virasoro algebra | 7 |
| Lecture 2 | 11 |
| 2.1. Definition of positive-energy representations of Vir | 11 |
| 2.2. Oscillator algebra \mathcal{A} | 12 |
| 2.3. Oscillator representations of Vir | 15 |
| Lecture 3 | 19 |
| 3.1. Complete reducibility of the oscillator representations of Vir | 19 |
| 3.2. Highest weight representations of Vir | 21 |
| 3.3. Verma representations $M(c, h)$ and irreducible highest weight representations $V(c, h)$ of Vir | 23 |
| 3.4. More (unitary) oscillator representations of Vir | 26 |
| Lecture 4 | 33 |
| 4.1. Lie algebras of infinite matrices | 33 |
| 4.2. Infinite wedge space F and the Dirac positron theory | 35 |
| 4.3. Representation of GL_∞ and $g\ell_\infty$ in F . Unitarity of highest weight representations of $g\ell_\infty$ | 38 |
| 4.4. Representation of α_∞ in F | 43 |
| 4.5. Representations of Vir in F | 45 |
| Lecture 5 | 49 |
| 5.1. Boson-fermion correspondence | 49 |
| 5.2. Wedging and contracting operators | 51 |
| 5.3. Vertex operators. The first part of the boson-fermion correspondence | 53 |
| 5.4. Vertex representations of $g\ell_\infty$ and α_∞ | 56 |

| | |
|---|-----|
| Lecture 6 | 59 |
| 6.1. Schur polynomials | 59 |
| 6.2. The second part of the boson-fermion correspondence | 61 |
| 6.3. An application: structure of the Virasoro representations for $c = 1$ | 64 |
| Lecture 7 | 69 |
| 7.1. Orbit of the vacuum vector under GL_∞ | 69 |
| 7.2. Defining equations for Ω in $F^{(0)}$ | 69 |
| 7.3. Differential equations for Ω in $\mathbb{C}[x_1, x_2, \dots]$ | 71 |
| 7.4. Hirota's bilinear equations | 72 |
| 7.5. KP hierarchy | 74 |
| 7.6. N -soliton solutions | 77 |
| Lecture 8 | 81 |
| 8.1. Degenerate representations and the determinant $\det_n(c, h)$ of the contravariant form | 81 |
| 8.2. The determinant $\det_n(c, h)$ as a polynomial in h | 83 |
| 8.3. The Kac determinant formula | 85 |
| 8.4. Some consequences of the determinant formula for unitarity and degeneracy | 88 |
| Lecture 9 | 93 |
| 9.1. Representations of loop algebras in $\bar{\mathcal{A}}_\infty$ | 93 |
| 9.2. Representations of \hat{gl}'_n in $F^{(m)}$ | 96 |
| 9.3. The invariant bilinear form on \hat{gl}_n . The action of \widetilde{GL}_n on \hat{gl}_n | 97 |
| 9.4. Reduction from a_∞ to \hat{sl}_n and unitarity of highest weight representations of \hat{sl}_n | 100 |
| Lecture 10 | 105 |
| 10.1. Nonabelian generalization of Virasoro operators: the Sugawara construction | 105 |
| 10.2. The Goddard-Kent-Olive construction | 113 |
| Lecture 11 | 117 |
| 11.1. \hat{sl}_2 and its Weyl group | 117 |
| 11.2. The Weyl-Kac character formula and Jacobi-Riemann theta functions | 119 |

| | |
|--|-----|
| 11.3. A character identity | 124 |
| Lecture 12 | 129 |
| 12.1. Preliminaries on $\hat{\mathfrak{sl}}_2$ | 129 |
| 12.2. A tensor product decomposition of some representations of $\hat{\mathfrak{sl}}_2$ | 130 |
| 12.3. Construction and unitarity of the discrete series representations of Vir | 132 |
| 12.4. Completion of the proof of the Kac determinant formula | 137 |
| 12.5. On non-unitarity in the region $0 < c < 1, h \geq 0$ | 138 |
| References | 141 |