CONTENTS

Chapter 1	
THE LIE ALGEBRA 9(A)	1
1.1 Realization of a Complex $n \times n$ Matrix A	1
1.2 Construction of the Lie Algebra 9(A)	3
1.3 Construction of the Lie Algebra g(A)(continued)	8
1.4 A Characterization of the Lie Algebra Q(A)	9
1.5 The Derived Algebra g'(A) of g(A)	12
1.6 Center of $g(A)$ and $g'(A)$	14
1.7 Minimal Number of Generators of 9(A)	15
1.8 The Subalgebra Associated with a Principal Submatrix	17
1.9 Decomposability	19
1.10 Some Simplicity Propositions	20
Chapter 2	
CLASSIFICATION OF GENERALIZED CARTAN	
MATRICES	24
2.1 A Fundamental Fact from the Theory of Linear Inequalities	24
2.2 Vinberg's Classification Theorem	26
2.3 Properties of Matrices of Finite and Affine Types	29
2.4 Properties of Generalized Cartan Matrix of Finite or Affine Types	31
2.5 Classification of Generalized Cartan Matrices of Finite and Affine Types	34
2.6 Classification of Generalized Cartan Matrices of Hyperbolic Type	39
Chapter 3	
THE INVARIANT BILINEAR FORM	52
3.1 The Existence of the Invariant Bilinear Form	52
3.2 Uniqueness of the Invariant Bilinear Form	58
3.3 The Case A Being a Symmetrizable Generalized Cartan Matrix	59
3.4 The Case A Being a Generalized Cartan Matrix of Affine Type	59

Chapter 4	
THE WEYL GROUP	63
	(2
4.1 The Relations Satisfied by the Chevalley Generators	63
4.2 The Weyl Group	65
4.3 The Tits Cone	71
4.4 The Case of A Being a Symmetrizable Generalized Cartan Matrix	77
4.5 Weight Strings	79
4.6 Characterization of Kac-Moody Algebra Q(A) Assosiated with	
a generalized Cartan Matrix of Finite Type	81
Chapter 5	
REAL AND IMAGINARY ROOTS	84
5.1 Definitions and Elementary Properties	84
5.2 Kac's Description of the Set of Imaginary Roots	86
5.3 Existence of Imaginary Roots	88
5.4 Description of the Set of Short Real, Long Real and Imaginary Roots	89
5.5 Root systems of Affine Lie Algebra	91
5.6 The Tits Cone and the Imaginary Cone	98
5.7 The Root Base	101
Chapter 6	
WEYL GROUPS OF AFFINE LIE ALGEBRAS	105
6.1 The Weyl Group of an Affine Lie Algebra	105
6.2 The Affine Weyl Group	112
Chapter 7	
REALIZATION OF AFFINE LIE ALGEBRAS	118
7.1 Realization of Non-twisted Affine Lie Algebras	118
7.2 Realization of Twisted Affine Lie Algebras	127

apter 8 FRODUCTION TO REPRESENTATION THEORY	
OF KAC-MOODY ALGEBRAS	141
3.1 $g(A)$ —modules, Category O and Formal Characters	141
3.2 Generalized Casimir Operator	147
3.3 Integrable Highest Weight Modules and Character Formula	153
Index	15