

CONTENTS

Introduction	vii
PART I: ALGEBRA	1
Chapter 1. An Informal Introduction	3
§1.1. “There is a spectral sequence”	3
§1.2. Lacunary phenomena	6
§1.3. Exploiting further structure	9
§1.4. Working backwards	20
§1.5. Interpreting the answer	23
Chapter 2. What is a Spectral Sequence?	27
§2.1. Definitions and basic properties	27
§2.2. How does a spectral sequence arise?	30
§2.2.1. Filtered differential modules	30
§2.2.2. The proof of theorem 2.1	33
§2.2.3. Exact couples	36
§2.2.4. The equivalence of the two approaches	41
§2.3. On convergence	43
Chapter 3. Tools and Examples	47
§3.1. Comparing spectral sequences	47
§3.1.1. Zeeman’s comparison theorem	55
§3.1.2. Spectral sequences of algebras	58
§3.2. Algebraic applications	62
§3.2.1. Double complexes	62
§3.2.2. The Künneth spectral sequence	66
Exercises	75
PART II: TOPOLOGY	81
Chapter 4. Topological Background	83
§4.1 CW-complexes	85
§4.1.1. Cofibrations	88
§4.1.2. Cellular homology	92
§4.2. Simplicial sets	95
§4.3 Fibrations	101
§4.3.1. Eilenberg–MacLane spaces and Postnikov towers	107
§4.4. Hopf algebras and the Steenrod algebra	112

Chapter 5. The Leray-Serre spectral sequence I	123
§5.1. Construction of the spectral sequence	125
§5.1.1. Multiplicative properties of the spectral sequence	128
§5.2 Immediate applications	129
§5.2.1 Particular fibrations	139
§5.2.2. Loops on a space	148
§5.3. Appendices	153
§5.3.1. Systems of local coefficients	154
§5.3.2. Proof of the main theorem	159
Chapter 6. The Leray-Serre spectral sequence II	165
§6.0. A proof of theorem 6.1	166
§6.1. The transgression	172
§6.1.1. Kudo's theorem	178
§6.1.2. On $H^*(K(\pi, n); \mathbf{Z}/p\mathbf{Z})$: the theorems of Cartan and Serre	181
§6.2. On classifying spaces and characteristic classes	192
§6.2.1. On vector bundles and classifying spaces	193
§6.2.2. On the cohomology of classifying spaces	198
§6.2.3. Applications	200
§6.3. Other constructions of the spectral sequence	205
§6.3.1. Bisimplicial sets and Dress' construction	208
Exercises	213
Chapter 7. The Eilenberg-Moore Spectral Sequence I	217
§7.1. Differential homological algebra	219
§7.1.1. The category of DG-modules over a DG-algebra	220
§7.1.2. Computations (the bar construction)	228
§7.2. Bringing in the topology	232
§7.2.1. The homology of quotient spaces of group actions	242
§7.3. Simple examples and a computational tool	248
Chapter 8. The Eilenberg-Moore Spectral Sequence II	253
§8.1. On homogeneous spaces	254
§8.1.1. Baum's thesis	257
§8.1.2. Another approach to differential homological algebra	265
§8.1.3. Extending the functor Tor	270
§8.2. Further structure	275
§8.2.1. The Eilenberg-Moore spectral sequence as the Künneth spectral sequence	276
§8.2.2. Cosimplicial techniques	283
§8.3. Differentials in the Eilenberg-Moore spectral sequence	287
§8.3.1. Massey's triple product	288

§8.3.2. Higher order Massey products	290
§8.3.3. Matric Massey products	295
Exercises	301
Chapter 9. The Adams Spectral Sequence	305
§9.1. Motivation: What cohomology sees	307
§9.1.1. Higher order cohomology operations	314
§9.2. More homological algebra: the functor Ext	314
§9.2.1. Multiplicative structure	318
§9.2.2. A change-of-rings spectral sequence	325
§9.3. The spectral sequence	330
§9.3.1. The construction: Adams resolutions	331
§9.3.2. The convergence	337
§9.3.3. Multiplicative structure	343
§9.4. Computations	348
§9.4.1. Low-dimensional calculations	348
§9.4.2. The first non-trivial differentials	358
§9.4.3. Massey products	359
§9.5. Further structure	364
§9.5.1. The vanishing line	364
§9.5.2. Periodicity	367
§9.5.3. The May spectral sequence	372
§9.5.4. Extensions and differentials	376
Exercises	382
PART III: SINS OF OMISSION	385
Chapter 10. Spectral sequences in Algebra, Algebraic Geometry and Algebraic K-theory	387
§10.1. The Lyndon-Hochschild-Serre spectral sequence	387
§10.2. Spectral sequences for rings and modules	388
§10.3. The Grothendieck spectral sequence	392
§10.4. Algebro-geometric constructions	393
§10.5. Spectral sequences in algebraic K-theory	395
Chapter 11. More Spectral Sequences in Topology	397
§11.1. The Bockstein spectral sequence	397
§11.2. Spectral sequences for mappings and spaces of mappings	399
§11.3. Spectral sequences and spectra	400
§11.4. Other Adams spectral sequences	403
§11.4.1. Unstable Adams spectral sequences	404
§11.5. Miscellanea	406
Bibliography	409
Index	419