

# Contents

List of frequently used notation	i
<b>1 Introduction and statement of results</b>	<b>1</b>
1.1 Historical background . . . . .	1
1.2 Statement of results . . . . .	4
1.3 Acknowledgements . . . . .	7
<b>2 Background material on categories of unstable modules and algebras</b>	<b>8</b>
2.1 Basic definitions and notation . . . . .	8
2.2 A Morita theorem for $K - \mathcal{U}$ . . . . .	11
2.3 The functors $\Sigma$ and $\mathcal{O}$ . . . . .	12
2.4 Nilpotent and reduced modules . . . . .	14
2.5 Free modules and Brown-Gitler modules in $K - \mathcal{U}$ . . . . .	16
<b>3 Lannes' <math>T</math>-functor and special injectives in <math>K - \mathcal{U}</math></b>	<b>23</b>
3.1 Review of the $T$ -functor – definitions and properties . . . . .	23
3.2 The functors $\text{Fix}_{(W,V)}$ . . . . .	29
3.3 A generalized linearization principle . . . . .	31
3.4 A generalized splitting theorem . . . . .	33
3.5 Some special injectives in $K - \mathcal{U}$ . . . . .	40
<b>4 The categories <math>\mathcal{S}(K)</math>, <math>\mathcal{R}(K)</math> and <math>\mathcal{F}(\mathcal{Y})</math> and some of their properties</b>	<b>47</b>
4.1 The categories $\mathcal{S}(K)$ and $\mathcal{R}(K)$ and invariant prime ideals in $K$ . . . . .	47
4.2 A description of morphisms in $\mathcal{S}(K)$ by matrices . . . . .	50
4.3 The category $\mathcal{F}(\mathcal{Y})$ and generalized polynomial functors . . . . .	53
<b>5 Locally noetherian categories</b>	<b>60</b>
5.1 Some category theoretical background material . . . . .	60
5.2 $K - \mathcal{U}$ is locally noetherian if $K$ is noetherian . . . . .	65

<b>6 Proof of the classification theorems for <math>K - \mathcal{U}</math> and <math>K_{fg} - \mathcal{U}</math></b>	<b>68</b>
6.1 The indecomposability of $E_K(\omega)$ . . . . .	68
6.1.1 The case $n = 0$ . . . . .	69
6.1.2 The case $n > 0$ . . . . .	72
6.2 The uniqueness of the parameters $\omega \in \Omega_K$ . . . . .	72
6.3 The completeness of $\{E_K(\omega)\}_{\omega \in \Omega_K}$ . . . . .	73
6.3.1 The torsion free case . . . . .	74
6.3.2 The general case . . . . .	78
6.4 A simpler description of $E_K(\omega)$ . . . . .	81
6.4.1 The functors $\tau_{(W, \psi)}$ . . . . .	81
6.4.2 The alternative description of $E_K(\omega)$ in the general case . . . . .	82
6.5 The classification theorem for $K_{fg} - \mathcal{U}$ . . . . .	84
<b>7 Applications</b>	<b>85</b>
7.1 A simple proof of Prop. 2.2 in [LZ3] . . . . .	85
7.2 Localizations in $K - \mathcal{U}$ away from torsion modules . . . . .	86
<b>8 Some examples</b>	<b>99</b>
8.1 Example: $K = H^*(D_{2^m}; \mathbb{F}_2)$ . . . . .	100
8.2 Example: $K = H^*(Q_{2^m}; \mathbb{F}_2)$ . . . . .	106
8.3 Example: $K = H^*(\Sigma_4; \mathbb{F}_2)$ . . . . .	107
8.4 Example: $K = D(m, s)$ . . . . .	110
<b>A Review of some algebraic facts and definitions</b>	<b>117</b>
<b>B Proof of Theorem 5.2.1</b>	<b>123</b>
<b>Bibliography</b>	<b>126</b>