

Contents

List of Tables	vii
List of Figures	xi
Introduction	xiii
Chapter 1. Quaternion Algebras and Quaternion Orders	1
1.1. Introduction to quaternion algebras	1
1.2. Introduction to quaternion orders	7
Chapter 2. Introduction to Shimura Curves	17
2.1. The Poincaré half-plane	17
2.2. Linear fractional transformations	18
2.3. Groups of quaternion transformations	22
2.4. The Shimura curves $X(D, N)$	24
Chapter 3. Quaternion Algebras and Quadratic Forms	27
3.1. Quadratic forms over rings	27
3.2. Principal forms	33
3.3. Quadratic forms associated to K -algebras	38
3.4. Quadratic forms associated to quadratic fields and orders	40
3.5. Normic forms of quaternion algebras	43
3.6. Normic forms of quaternion orders	49
Chapter 4. Embeddings and Quadratic Forms	55
4.1. Embeddings of quadratic fields into quaternion algebras	55
4.2. Embeddings of quadratic orders into quaternion orders	57
4.3. Classification of representations by normic ternary forms	61
4.4. Binary forms associated to quaternion orders	67
4.5. Classification of binary forms associated to quaternion orders	71
Chapter 5. Hyperbolic Fundamental Domains for Shimura Curves	77
5.1. Fundamental domains of modular curves	77
5.2. Transformations, embeddings and forms	79
5.3. Local conditions at infinity	86
5.4. Principal symmetries of $\Gamma(D, N)$	89
5.5. Construction of fundamental domains ($D > 1$)	91
Chapter 6. Complex Multiplication Points in Shimura Curves	103
6.1. Classification of complex multiplication points	103
6.2. Special complex multiplication points	108
6.3. Computation of complex multiplication points	109

6.4. Examples in the nonramified case	112
6.5. Examples in the small ramified case	120
Chapter 7. The Poincaré Package	123
7.1. Main characteristics	123
7.2. Description of the instructions	124
7.3. List of instructions	130
Appendix A. Tables	145
A.1. Quaternion algebras	145
A.2. Shimura curves	153
A.3. Quaternion algebras and quadratic forms	160
A.4. Embeddings and quadratic forms	165
A.5. Complex multiplication points in Shimura curves	183
Appendix B. Further Contributions to the Study of Shimura Curves	187
Appendix C. Applications of Shimura Curves	189
Bibliography	191
Index	195