CONTENTS

Preface vii

Chapter One. Affine Algebraic Sets

1. Algebraic Preliminaries 1
2. Affine Space and Algebraic Sets 7
3. The Ideal of a Set of Points 10
4. The Hilbert Basis Theorem 13
5. Irreducible Components of an Algebraic Set 15
6. Algebraic Subsets of the Plane 18
7. Hilbert's Nullstellensatz 20
8. Modules; Finiteness Conditions 25
9. Integral Elements 28
10. Field Extensions 31

Chapter Two. Affine Varieties

1. Coordinate Rings 35
2. Polynomial Maps 37
3. Coordinate Changes 40
4. Rational Functions and Local Rings 42
5. Discrete Valuation Rings 46
6. Forms 50
7. Direct Products of Rings 52
8. Operations with Ideals 53
9. Ideals with a Finite Number of Zeros 55
10. Quotient Modules and Exact Sequences 57
11. Free Modules 61

Chapter Three. Local Properties of Plane Curves

1. Multiple Points and Tangent Lines 63

2. Multiplicities and Local Rings 70
3. Intersection Numbers 74

Chapter Four. Projective Varieties

1. Projective Space 85
2. Projective Algebraic Sets 88
3. Affine and Projective Varieties 96
4. Multiprojective Space 100

Chapter Five. Projective Plane Curves

1. Definitions 103
2. Linear Systems of Curves 108
3. Bezout's Theorem 112
4. Multiple Points 117
5. Max Noether's Fundamental Theorem 119
6. Applications of Noether's Theorem 123

Chapter Six. Varieties, Morphisms, and Rational Maps

1. The Zariski Topology 131
2. Varieties 135
3. Morphisms of Varieties 137
4. Products and Graphs 144
5. Algebraic Function Fields and Dimension of Varieties 149
6. Rational Maps 153

Chapter Seven. Resolution of Singularities

1. Rational Maps of Curves 159
2. Blowing up a point in A^2 162
3. Blowing up points in P^2 168
4. Quadratic Transformations 171

5. Non-singular Models of Curves 179

Chapter Eight. Riemann-Roch Theorem

1. Divisors 187
2. The Vector Spaces L(D) 192
3. Riemann's Theorem 196
4. Derivations and Differentials 203
5. Canonical Divisors 206
6. Riemann-Roch Theorem 209

Appendix. Non-zero Characteristic 219

Suggestions for Further Reading 221

Notation 223

Index 225