

Table of Contents

Introduction.....	vii
Prerequisites	ix
Chapter 1. Affine Varieties	1
§1A. Their Definition, Tangent Space, Dimension, Smooth and Singular Points	1
§1B. Analytic Uniformization at Smooth Points, Examples of Topological Knottedness at Singular Points	9
§1C. $\mathcal{O}_{x,X}$ a UFD when x Smooth; Divisor of Zeroes and Poles of Functions	14
Chapter 2. Projective Varieties	20
§2A. Their Definition, Extension of Concepts from Affine to Projective Case	20
§2B. Products, Segre Embedding, Correspondences	26
§2C. Elimination Theory, Noether's Normalization Lemma, Density of Zariski-Open Sets	33
Chapter 3. Structure of Correspondences	40
§3A. Local Properties—Smooth Maps, Fundamental Openness Principle, Zariski's Main Theorem	40
§3B. Global Properties—Zariski's Connectedness Theorem, Specialization Principle.....	49
§3C. Intersections on Smooth Varieties.....	56
Chapter 4. Chow's Theorem.....	59
§4A. Internally and Externally Defined Analytic Sets and their Local Descriptions as Branched Coverings of \mathbb{C}^n	59
§4B. Applications to Uniqueness of Algebraic Structure and Connectedness.....	67
Chapter 5. Degree of a Projective Variety	70
§5A. Definition of $\deg X$, $\text{mult}_x X$, of the Blow up $B_x(X)$, Effect of a Projection, Examples	70

§5B. Bezout's Theorem	80
§5C. Volume of a Projective Variety; Review of Homology, DeRham's Theorem, Varieties as Minimal Submanifolds.....	85
Chapter 6. Linear Systems	96
§6A. The Correspondence between Linear Systems and Rational Maps, Examples; Complete Linear Systems are Finite-Dimensional.....	96
§6B. Differential Forms, Canonical Divisors and Branch Loci.....	104
§6C. Hilbert Polynomials, Relations with Degree	110
Appendix to Chapter 6. The Weil-Samuel Algebraic Theory of Multiplicity	116
Chapter 7. Curves and Their Genus	127
§7A. Existence and Uniqueness of the Non-Singular Model of Each Function Field of Transcendence Degree 1 (after Albanese).....	127
§7B. Arithmetic Genus = Topological Genus; Existence of Good Projections to \mathbb{P}^1 , \mathbb{P}^2 , \mathbb{P}^3	131
§7C. Residues of Differentials on Curves, the Classical Riemann-Roch Theorem for Curves and Applications.....	142
§7D. Curves of Genus 1 as Plane Cubics and as Complex Tori \mathbb{C}/L	149
Chapter 8. The Birational Geometry of Surfaces	156
§8A. Generalities on Blowing up Points	156
§8B. Resolution of Singularities of Curves on a Smooth Surface by Blowing up the Surface; Examples.....	160
§8C. Factorization of Birational Maps between Smooth Surfaces; the Trees of Infinitely Near Points	168
§8D. The Birational Map between \mathbb{P}^2 and the Quadric and Cubic Surfaces; the 27 Lines on a Cubic Surface.....	172
Bibliography	181
List of Notations	183
Index	184