

Contents

Acknowledgment	xv
Some Standard Notation	xvii
CHAPTER 1	
Absolute Values	1
1. Definitions, dependence and independence	1
2. Completions	5
3. Unramified extensions	9
4. Finite extensions	12
CHAPTER 2	
Proper Sets of Absolute Values.	
Divisors and Units	18
1. Proper sets of absolute values	18
2. Number fields	19
3. Divisors on varieties	21
4. Divisors on schemes	24
5. M_K -divisors and divisor classes	29
6. Ideal classes and units in number fields	32
7. Relative units and divisor classes	41
8. The Chevalley–Weil theorem	44
CHAPTER 3	
Heights	50
1. Definitions	50
2. Gauss' lemma	54
3. Heights in function fields	62

4. Heights on abelian groups	66
5. Counting points of bounded height	70
CHAPTER 4	
Geometric Properties of Heights	76
1. Functorial properties	76
2. Heights and linear systems	83
3. Ample linear systems	87
4. Projections on curves	90
5. Heights associated with divisor classes	91
CHAPTER 5	
Heights on Abelian Varieties	95
1. Some linear and quasi-linear algebra	95
2. Quadraticity of endomorphisms on divisor classes	99
3. Quadraticity of the height	106
4. Heights and Poincaré divisors	110
5. Jacobian varieties and curves	113
6. Definiteness properties Over number fields	120
7. Non-degenerate heights and Euclidean spaces	124
8. Mumford's theorem	134
CHAPTER 6	
The Mordell–Weil Theorem	138
1. Kummer theory	139
2. The weak Mordell–Weil theorem	144
3. The infinite descent	145
4. Reduction steps	146
5. Points of bounded height	149
6. Theorem of the base	153
CHAPTER 7	
The Thue–Siegel–Roth Theorem	158
1. Statement of the theorem	158
2. Reduction to simultaneous approximations	163
3. Basic steps of the proof	165
4. A combinatorial lemma	170
5. Proof of Proposition 3.1	171
6. Wronskians	173
7. Factorization of a polynomial	175
8. The index	178
9. Proof of Proposition 3.2	181
10. A geometric formulation of Roth's theorem	183

CHAPTER 8	
Siegel's Theorem and Integral Points	188
1. Height of integral points	189
2. Finiteness theorems	192
3. The curve $ax + by = 1$	194
4. The Thue–Siegel curve.	196
5. Curves of genus 0.	197
6. Torsion points on curves.	200
7. Division points on curves	205
8. Non-abelian Kummer theory.	212
CHAPTER 9	
Hilbert's Irreducibility Theorem	225
1. Irreducibility and integral points	226
2. Irreducibility Over the rational numbers	229
3. Reduction steps	233
4. Function fields	236
5. Abstract definition of Hilbert sets	239
6. Applications to commutative group varieties	242
CHAPTER 10	
Weil Functions and Néron Divisors	247
1. Bounded sets and functions	247
2. Néron divisors and Weil functions	252
3. Positive divisors	258
4. The associated height function	263
CHAPTER 11	
Néron Functions on Abelian Varieties	266
1. Existence of Néron functions	266
2. Translation properties of Néron functions	271
3. Néron functions on varieties	276
4. Reciprocity laws	283
5. Néron functions as intersection multiplicities	286
6. The Néron symbol and group extensions	290
CHAPTER 12	
Algebraic Families of Néron Functions	296
1. Variation of Néron functions in an algebraic family	297
2. Silverman's height and specialization theorems	303
3. Néron heights as intersection multiplicities	307
4. Fibral divisors	314
5. The height determined by a section: Tate's theorem	320

CHAPTER 13

Néron Functions Over the Complex Numbers	324
1. The Néron function of an abelian variety	324
2. The scalar product of differentials of first kind	327
3. The canonical 2-form and the Riemann theta function	332
4. The divisor of the Riemann theta function	334
5. Green, Néron, and theta functions	339
6. The law of interchange of argument and parameter	341
7. Differentials of third kind and Green's function	344
Appendix	347
Review of S. Lang's <i>Diophantine Geometry</i> , by L. J. Mordell	349
Review of L. J. Mordell's <i>Diophantine Equations</i> , by S. Lang	355
Bibliography	359
Index	367