

Contents

Introduction	1
Chapter 1. Breaks and Swan Conductors	12
1.0 The basic setting	12
1.1–1.10 Definitions and basic properties of breaks, break-decompositions, and Swan conductors	12
1.11–1.17 Representation-theoretic consequences of particular arrays of breaks	17
1.18–1.20 Detailed analysis when $\text{Swan} = 1$	23
Chapter 2. Curves and Their Cohomology	26
2.0 Generalities	26
2.1 Cohomology of wild sheaves	28
2.2 Canonical calculations of cohomology	28
2.3 The Euler–Poincaré and Lefschetz trace formulas	31
Chapter 3. Equidistribution in Equal Characteristic	36
3.0–3.5 The basic setting	36
3.6 The equidistribution theorem	38
3.7 Remark on the integration of sufficiently smooth functions	42
Chapter 4. Gauss Sums and Kloosterman Sums: Kloosterman Sheaves	46
4.0 Kloosterman sums as inverse Fourier transforms of monomials in Gauss sums	46
4.1 The existence theorem for Kloosterman sheaves	48
4.2 Signs of pairings	54
4.3 The existence theorem for $n = 1$, via the sheaves \mathcal{L}_ψ and \mathcal{L}_χ	59

Chapter 5. Convolution of Sheaves on G_m	62
5.0 The basic setting, and a lemma	62
5.1 Statement of the convolution theorem	63
5.2 Proof of the convolution theorem and variations	65
5.3 Convolution and duality: Signs	77
5.4 Multiple convolution	79
5.5 First applications to Kloosterman sheaves	80
5.6 Direct images of Kloosterman sheaves, via Hasse– Davenport	84
Chapter 6. Local Convolution	87
6.0 Formulation of the problem	87
6.1 Consequences of a solution	87
6.2–6.4 Preparations	88
6.5 A theorem on vanishing cycles	90
6.6 Construction of local convolution	95
Chapter 7. Local Monodromy at Zero of a Convolution: Detailed Study	96
7.0 General review of local monodromy	96
7.1 Application to a “product formula” for a convolution of pure sheaves	100
7.2 Application to sheaves with $\text{Swan}_\infty = 1$	104
7.3 Application to Kloosterman sheaves	105
7.4 Some special cases	107
7.5 Appendix: The product formula in the general case (d’après O. Gabber)	109
7.6 Appendix: An open problem concerning breaks of a convolution	117
Chapter 8. Complements on Convolution	120
8.0 A cancellation theorem for convolution	120
8.1 Two variants of the cancellation theorem	127
8.2 Interlude: Naive Fourier transform	130
8.3 Basic examples of Fourier sheaves	134
8.4 Irreducible Fourier sheaves	135
8.5 Numerology of Fourier transform	136
8.6 Convolution with \mathcal{L}_ψ as Fourier transform	142

8.7 Ubiquity of Kloosterman sheaves	146
8.8 The structure over \mathbf{F}_q : Canonical descents of Kloosterman sheaves	148
8.9 Embedding in a compatible system	152
Chapter 9. Equidistribution in $(S^1)^r$ of r-tuples of Angles of Gauss Sums	155
9.0 Motivation: Davenport's theorem on consecutive quadratic residues	155
9.1–9.2 Formulation of the problem	157
9.3–9.5 The theorem, a reformulation, and a reduction	158
9.7 The end of the proof	165
9.8 Remark concerning the proof	166
Chapter 10. Local Monodromy at ∞ of Kloosterman Sheaves	168
10.0 Review of some "independence of l " results	168
10.1 Independence of l and χ 's as P_∞ -representation	168
10.2 Independence of l and χ 's of ∞ -breaks of tensor products	169
10.3–10.4 Breaks and Swans of tensor products	169
Chapter 11. Global Monodromy of Kloosterman Sheaves	176
11.0 Formulation of the theorem	176
11.1 Statement of the main theorem	178
11.2 Remarks and comments	179
11.3 A corollary	179
11.4 First application to equidistribution	180
11.5 Axiomatics of the proof; classification theorems	180
11.6 The classification theorem	184
11.7 An axiomatic classification theorem	185
11.8 G_2 Theorem	186
11.9 Density Theorem	186
11.10 Proof of the classification theorem (11.6)	186
11.11 Proof of the G_2 Theorem	200

Chapter 12. Integral Monodromy of Kloosterman Sheaves (d'après O. Gabber)	210
12.0 Formulation of the theorem	210
12.1 The theorem	211
12.2 A more precise version	212
12.3 Reduction to a universal situation	212
12.4 Generation by unipotent elements	217
12.5 Analysis of the special case $p = 2$, n odd $\neq 7$	227
12.6 Analysis of the special case $p = 2$, $n = 7$	229
Chapter 13. Equidistribution of “Angles” of Kloosterman Sums	234
13.0–13.4 Uniform description of the space of conjugacy classes and its Haar measure	234
13.5 Formulation of the theorem	238
13.6 Example: the simplest case	240
References	243