

Contents

§0. Waffle	1
Reasons for studying algebraic geometry, the 'subset' problem; different categories of geometry, need for commutative algebra, partially defined function; character of the author. Prerequisites, relations with other courses, list of books.	
Chapter I. Playing with plane curves	9
§1. Plane conics	9
General familiarity with \mathbb{P}^2 and homogeneous coordinates, relation of \mathbb{A}^2 to \mathbb{P}^2 ; parametrisation, every smooth conic $C \subset \mathbb{P}^2$ is $\cong \mathbb{P}^1$. Easy cases of Bézout's theorem: line \cap curve of degree $d = d$ points, conic \cap curve of degree $d = 2d$ points; linear system of conics through P_1, \dots, P_n .	
§2. Cubics and the group law	27
The curve $(y^2 = x(x-1)(x-\lambda))$ has no rational parametrisation. Linear systems $S_d(P_1, \dots, P_n)$; pencil of cubics through 8 points 'in general position'; group law on cubic; Pascal's mystic hexagon.	
Appendix to Chapter I. Curves and their genus	43
Topology of non-singular plane cubics over \mathbb{C} ; informal discussion of the genus of a curve; topology, differential geometry, moduli, number theory, Mordell-Weil-Faltings.	
Chapter II. The category of affine varieties	48
§3. Affine varieties and the Nullstellensatz	48
Noetherian rings, Hilbert Basis Theorem; correspondences V and I , irreducible algebraic sets, Zariski topology, statement of Nullstellensatz; irreducible hypersurface. Noether normalisation and proof of Nullstellensatz; reduction to a hypersurface.	
§4. Functions on varieties	66
Coordinate ring and polynomial maps; morphisms and isomorphisms; affine varieties. Rational function field and rational maps; dominant rational maps, and composing rational maps; standard open sets; addition law on elliptic curve is a morphism.	

Chapter III. Applications	79
§5. Projective varieties and birational equivalence	79
<p>Motivation: there are varieties strictly bigger than any affine variety; homogeneous V-I correspondences; projective versus affine. Examples: quadric surfaces; Veronese surface. Birational equivalence, rational varieties; every variety is birational to a hypersurface; products.</p>	
§6. Tangent space and non-singularity, dimension	94
<p>Motivation: implicit function theorem, varieties and manifolds. Definition of affine tangent space; non-singular points are dense. Tangent space and m/m^2, tangent space is intrinsic; dimension of $X = \text{tr deg}_k k(X)$. Resolution of singularities by blow-ups.</p>	
§7. 27 lines on cubic surface	102
<p>The cubic surface is rational and has 27 lines forming a classical configuration; the case when all lines are real.</p>	
§8. Final comments	112
<p>History and sociology. Choice of topics, high-brow remarks and technical notes. Substitute for preface; acknowledgements and name-dropping.</p>	
Index	127