

Contents

Preface	vii
List of tables	xxi
<i>Chapter 1</i>	
General background results	1
1.1 Some basic definitions	1
1.2 Surface singularities	12
1.3 On the singularities that arise in adjunction theory	16
1.4 Curves	21
1.5 Nefvalue results	25
1.6 Universal sections and discriminant varieties	27
1.7 Bertini theorems	34
1.8 Some examples	40
<i>Chapter 2</i>	
Consequences of positivity	42
2.1 k -ampleness and k -bigness	43
2.2 Vanishing theorems	45
2.3 The Lefschetz hyperplane section theorem	50
2.4 The Albanese mapping in the presence of rational singularities	56
2.5 The Hodge index theorem and the Kodaira lemma	58
2.6 Rossi's extension theorems	63
2.7 Theorems of Andreotti-Grauert and Griffiths	68
<i>Chapter 3</i>	
The basic varieties of adjunction theory	70
3.1 Recognizing projective spaces and quadrics	70
3.2 \mathbb{P}^d -bundles	73
3.3 Special varieties arising in adjunction theory	80

Chapter 4

The Hilbert scheme and extremal rays	83
4.1 Flatness, the Hilbert scheme, and limited families	83
4.2 Extremal rays and the cone theorem	90
4.3 Varieties with nonnef canonical bundle	98

Chapter 5

Restrictions imposed by ample divisors	103
5.1 On the behavior of k -big and ample divisors under maps	103
5.2 Extending morphisms of ample divisors	107
5.3 Ample divisors with trivial pluricanonical systems	111
5.4 Varieties that can be ample divisors only on cones	112
5.5 \mathbb{P}^d -bundles as ample divisors	117

Chapter 6

Families of unbreakable rational curves	120
6.1 Examples	121
6.2 Families of unbreakable rational curves	123
6.3 The nonbreaking lemma	126
6.4 Morphisms of varieties covered by unbreakable rational curves	136
6.5 The classification of projective manifolds covered by lines	144
6.6 Some spannedness results	148

Chapter 7

General adjunction theory	154
7.1 Spectral values	156
7.2 Polarized pairs $(\mathcal{M}, \mathcal{L})$ with nefvalue $> \dim \mathcal{M} - 1$ and \mathcal{M} singular	159
7.3 The first reduction of a singular variety	168
7.4 The polarization of the first reduction	173
7.5 The second reduction in the smooth case	176
7.6 Properties of the first and the second reduction	185
7.7 The second reduction (X, \mathcal{D}) with $K_X + (n - 3)\mathcal{D}$ nef	192
7.8 The three dimensional case	202
7.9 Applications	204

Chapter 8

Background for classical adjunction theory	213
8.1 Numerical implications of nonnegative Kodaira dimension	213
8.2 The double point formula for surfaces	217
8.3 Smooth double covers of irreducible quadric surfaces	217
8.4 Surfaces with one dimensional projection from a line	218
8.5 k -very ampleness	225
8.6 Surfaces with Castelnuovo curves as hyperplane sections	229
8.7 Polarized varieties (X, L) with sectional genus $g(L) = h^1(\mathcal{O}_X)$	234
8.8 Spannedness of $K_X + (\dim X)L$ for ample and spanned L	236
8.9 Polarized varieties (X, L) with sectional genus $g(L) \leq 1$	241
8.10 Classification of varieties up to degree 4	243

Chapter 9

The adjunction mapping	246
9.1 Spannedness of adjoint bundles at singular points	247
9.2 The adjunction mapping	249

Chapter 10

Classical adjunction theory of surfaces	253
10.1 When the adjunction mapping has lower dimensional image	254
10.2 Surfaces with sectional genus $g(L) \leq 3$	258
10.3 Very ampleness of the adjoint bundle	265
10.4 Very ampleness of the adjoint bundle for degree $d \geq 9$	265
10.5 Very ampleness of the adjoint bundle when $h^1(\mathcal{O}_S) > 0$	270
10.6 Very ampleness of the adjoint bundle when $h^1(\mathcal{O}_S) = 0$	273
10.7 Preservation of k -very ampleness under adjunction	278

Chapter 11

Classical adjunction theory in dimension ≥ 3	280
11.1 Some results on scrolls	281
11.2 The adjunction mapping with a lower dimensional image	284
11.3 Very ampleness of the adjoint bundle	287
11.4 Applications to hyperelliptic curve sections	288
11.5 Projective normality of adjoint bundles	292
11.6 Manifolds of sectional genus ≤ 4	294
11.7 The Fano-Morin adjunction process	298

Chapter 12

The second reduction in dimension three	302
12.1 Exceptional divisors of the second reduction morphism	303
12.2 The structure of the second reduction	308
12.3 The second reduction for threefolds in \mathbb{P}^5	314

Chapter 13

Varieties (M, L) with $\kappa(K_M + (\dim M - 2)L) \geq 0$	316
13.1 The double point formula for threefolds	317
13.2 The linear system $ K_M + (n - 2)L $ on the first reduction (M, L)	322
13.3 Some Chern inequalities for ample divisors	326

Chapter 14

Special varieties	328
14.1 Structure results for scrolls	329
14.2 Structure results for quadric fibrations	336
14.3 Varieties with small invariants	340
14.4 Projective manifolds with positive defect	346
14.5 Hyperplane sections of curves	351

Bibliography	355
Index	395