Contents

Preface ix

1. Construction of Drinfeld modular varieties 1

(1.0) Notations 1
(1.1) Endomorphisms of the additive group 1
(1.2) Drinfeld modules 3
(1.3) Level structures 5
(1.4) Modular varieties 7
(1.5) Deformation theory 9
(1.6) Hecke algebras, correspondences 12
(1.7) Hecke operators 15
(1.8) Comments and references 18

2. Drinfeld A-modules with finite characteristic 19

(2.0) Notations 19
(2.1) Isogenies 19
(2.2) Isogeny classes of Drinfeld modules 22
(2.3) Tate modules of a Drinfeld module 27
(2.4) Dieudonné modules 31
(2.5) Dieudonné module of a Drinfeld module 35
(2.6) First description of an isogeny class 43
(2.7) Isogeny classes as double coset spaces 47
(2.8) Comments and references 50

3. The Lefschetz numbers of Hecke operators 51

(3.0) Introduction 51

(3.1) The Lefschetz numbers of correspondences 51
(3.2) Counting of fixed points 52
(3.3) Where the orbital integrals come in 56
(3.4) Transfer of conjugacy classes 60
(3.5) Transfer of Haar measures 66
(3.6) The Lefschetz numbers as sums of twisted orbital integrals 72
(3.7) Comments and references 74

4. The fundamental lemma 75

(4.0) Introduction 75
(4.1) Satake isomorphism 76
(4.2) Base change homomorphism 83
(4.3) Orbital integrals 87
(4.4) Twisted orbital integrals 93
(4.5) Main theorem 98
(4.6) The elliptic case 102
(4.7) The general case 114
(4.8) Non-closed orbital integrals 117
(4.9) Comments and references 128

5. Very cuspidal Euler–Poincaré functions 129

(5.0) Introduction 129
(5.1) The function f 130
(5.2) Kottwitz's functions 133
(5.3) Elliptic orbital integrals of f 135
(5.4) K-invariant constant terms of f 142
(5.5) The function f is very cuspidal 152
(5.6) Non-elliptic orbital integrals of f 156
(5.7) Comments and references 156

6. The Lefschetz numbers as sums of global elliptic orbital integrals 158

7. Unramified principal series representations 160

(7.0) Introduction 160
(7.1) Parabolic induction and restriction 160
(7.2) Cuspidal representations 165
(7.3) Principal series representations 168
(7.4) Unramified principal series representations 178
(7.5) Spherical representations 187
(7.6) Comments and references 191

8. Euler–Poincaré functions as pseudocoefficients of the Steinberg representation 192

(8.0) Introduction 192
(8.1) The Steinberg representation 192
(8.2) Main theorem 207
(8.3) Some easy vanishing results 208
(8.4) Cohomological interpretation of $\mathrm{tr}\pi(f)$ 213
(8.5) Unitarizable representations 220
(8.6) Proof of Howe and Moore's criterion of non-unitarizability 226
(8.7) Comments and references 248

Appendices

A. Central simple algebras 249

(A.0) Central simple algebras 249
(A.1) Bicommutant theorem 250
(A.2) Central simple algebras over local fields 251
(A.3) Central simple algebras over function fields 253
(A.4) Comments and references 255

B. Dieudonné's theory : some proofs 256

(B.1) Proof of (2.4.5) 256
(B.2) Proof of (2.4.6) 268
(B.3) Proof of (2.4.11) 272
(B.4) Comments and references 280

C. Combinatorial formulas 281

(C.0) Introduction 281
(C.1) q-binomial coefficients 281

D. Representations of unimodular, locally compact, totally discontinuous, separated, topological groups 284

(D.0) Introduction 284
(D.1) Smooth representations of H 284
(D.2) Admissible representations of H 290
(D.3) Induction and restriction 292
(D.4) Cuspidal representations of H 311
(D.5) Injective and projective objects in $\mathrm{Rep}_s(H)$; cohomology 322
(D.6) Unitarizable representations 327
(D.7) Decomposition of representations into tensor products 334
(D.8) Comments and references 336

References 337

Index 341