

Contents

Preface	xiii
Description of the chapters	1
Chapter I. Integral closure	5
1. Introduction	5
2. Integral elements	9
3. Products of ideals	16
4. Noetherian rings	20
5. Rings of dimension 1	28
6. Dedekind domains	31
Chapter II. Plane curves	35
1. Introduction	35
2. Rings of functions	38
3. Points and maximal ideals	45
4. Morphisms of curves	47
5. Singular points	51
6. Localization	57
7. More on dimension	63
8. Local principal ideal domains	67
9. Localization of modules	71
10. Hilbert's Basis Theorem	76
11. More rings of functions	77
Chapter III. Factorization of ideals	85
1. Introduction	85
2. Unique factorization of ideals	88
3. Ramification index and residual degree	93
4. Explicit factorizations	98
5. Ramified and unramified primes	101

6. Simple extensions	105
7. Examples	108
8. Galois extensions	116
9. Galois covers	120
Chapter IV. The discriminants	131
1. Introduction	131
2. The discriminant as a norm	133
3. The discriminant of a basis	138
4. Examples of non-simple extensions	141
5. The discriminant ideal	143
6. Norm map on ideals	149
Chapter V. The ideal class group	157
1. Introduction	157
2. Definition of the ideal class group	158
3. Rings with finite quotients	160
4. The case of number fields	163
5. The case of function fields (I)	167
6. Absolute values and valuations	168
7. Archimedean absolute values and the product formula	172
8. The case of function fields (II)	176
9. Valuations and local principal ideal domains	181
10. Nonsingular complete curves	183
Chapter VI. Projective curves	193
1. Introduction	193
2. Projective spaces	194
3. Plane projective curves	199
4. Projective transformations	203
5. Conics	205
6. Projections	206
7. The tangent line at a point of a projective curve	209
8. Functions on a projective curve	213
9. Projective curves and valuations	217
10. The intersection of two projective curves	221
Chapter VII. Nonsingular complete curves	225
1. Introduction	225
2. Nonsingular curves and Dedekind domains, revisited	227
3. Fields of definition and Galois actions on curves	230
4. Function fields	236
5. Morphisms of nonsingular complete curves	244
6. Fields of definition, revisited	252

7. The divisor class group	259
Chapter VIII. Zeta-functions	269
1. Introduction	269
2. The Riemann ζ -function	274
3. ζ -functions and Euler products	276
4. Power series	277
5. The zeta-function of a nonsingular curve	279
6. The rationality of the zeta-function	284
7. The functional equation	288
8. Jacobi sums	292
9. Relations between class numbers	296
Chapter IX. The Riemann-Roch Theorem	305
1. Introduction	305
2. Laurent expansions	308
3. Riemann's Theorem	310
4. Duality	316
5. Changing the ground field	323
6. The genus of a nonsingular plane curve	327
7. The arithmetical genus	329
8. The Riemann-Hurwitz formula	331
9. Maps to projective spaces	333
Chapter X. Frobenius morphisms and the Riemann hypothesis	339
1. Inseparable extensions	339
2. The Frobenius morphisms	344
3. The Frobenius endomorphism	348
4. The Frobenius element at a point of a Galois cover	351
5. The Riemann hypothesis	354
6. End of the proof	358
Chapter XI. Further topics	361
1. Diophantine problems	361
2. Surfaces	362
3. The jacobian variety	363
4. Galois representations	365
5. The characteristic polynomial of Frobenius	366
6. From $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ to $\text{Gal}(\overline{\mathbb{F}}_p/\mathbb{F}_p)$	368
7. ζ -functions	371
8. Extensions with given Galois groups	373
Chapter XII. Appendix	375
1. Gauss' Lemma	375